

REPORT DOCUMENTATION PAGE

AFRL-SR-AR-TR-02-

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering the required information, reviewing and collecting the information, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Project, Washington, DC 20503.

wing
ation

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE

26-NOV-2002

3. REPORT TYPE AND DATES COVERED

FINAL (01-JUN-2001 TO 30-NOV-2001)

4. TITLE AND SUBTITLE

MID-FREQUENCY DYNAMICS OF COMPLEX STRUCTURAL SYSTEM:
ASSESSING THE STATE OF THE ART AND DEFINING FUTURE RESEARCH
DIRECTIONS.

5. FUNDING NUMBERS

F49620-01-1-0413

6. AUTHOR(S)

PROFESSOR CHRISTOPHER PIERRE

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

DEPARTMENT OF MECHANICAL ENGINEERING
2250 G. G. BROWN BUILDING
THE UNIVERSITY OF MICHIGAN
ANN ARBOR, MI 48109-2125

8. PERFORMING ORGANIZATION
REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR/NA
4015 WILSON BOULEVARD
ARLINGTON, VA 22203

10. SPONSORING/MONITORING
AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT

Approved for public release;
distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

One of the last frontiers of structural dynamics is mid-frequency vibration analysis of complex structures. In the low-frequency range, finite element analysis (FEA) is well established as the standard method. However, as the frequency of vibration increases, the cost of FEA becomes prohibitive due to the necessary refinement of the finite element mesh to capture the shorter wavelength of vibration. Furthermore, the system response becomes sensitive to small parameter variations at higher frequencies, which means that a statistical analysis should be employed to make confident response predictions. In the high-frequency range, statistical energy analysis (SEA) is popular. However, SEA provides only averaged response predictions and cannot capture the resonant behavior in the response that becomes evident as frequency decreases. Thus, there exists a mid-frequency range in which there is no established analysis technique analogous to FEA or SEA. The goal of this research is to produce a review paper on mid-frequency vibration analysis that will provide a survey of the relevant literature, identify the key technical challenges, formulate an assessment of the state of the art, and propose directions for future research.

14. SUBJECT TERMS

20030115 102

15. NUMBER OF PAGES

66

16. PRICE CODE

17. SECURITY CLASSIFICATION
OF REPORT

UNCLASSIFIED

18. SECURITY CLASSIFICATION
OF THIS PAGE

UNCLASSIFIED

19. SECURITY CLASSIFICATION
OF ABSTRACT

UNCLASSIFIED

20. LIMITATION OF ABSTRACT

Final Report to

NOV 26 2002

Air Force Office of Scientific Research
Aerospace and Materials Science
Structural Mechanics

AFOSR GRANT F49620-01-1-0413

Mid-Frequency Dynamics of Complex Structural Systems:
Assessing the State of the Art and Defining Future Research Directions

Christophe Pierre

Professor

Phone: (734) 936-0401 Fax: (734) 647-3170 Email: pierre@umich.edu

Matthew P. Castanier

Associate Research Scientist

Phone: (734) 647-5526 Fax: (734) 647-3170 Email: mpc@umich.edu

Department of Mechanical Engineering
2250 G. G. Brown Building
The University of Michigan
Ann Arbor, MI 48109-2125

ABSTRACT

One of the last frontiers of structural dynamics is mid-frequency vibration analysis of complex structures. In the low-frequency range, finite element analysis (FEA) is well established as the standard method. However, as the frequency of vibration increases, the cost of FEA becomes prohibitive due to the necessary refinement of the finite element mesh to capture the shorter wavelength of vibration. Furthermore, the system response becomes sensitive to small parameter variations at higher frequencies, which means that a statistical analysis should be employed to make confident response predictions. In the high-frequency range, statistical energy analysis (SEA) is popular. However, SEA provides only averaged response predictions and cannot capture the resonant behavior in the response that becomes evident as frequency decreases. Thus, there exists a mid-frequency range in which there is no established analysis technique analogous to FEA or SEA. The goal of this research is to produce a review paper on mid-frequency vibration analysis that will provide a survey of the relevant literature, identify the key technical challenges, formulate an assessment of the state of the art, and propose directions for future research.

OBJECTIVES

The main objectives of this research are as follows:

1. Perform a comprehensive literature search on mid-frequency vibration analysis and related topics, and collect an extensive set of references
2. Review the relevant literature and summarize the mid-frequency approaches and techniques that have been developed to date
3. Assess the state of the art by determining the major research challenges in the field and drawing conclusions on research progress to date, including applications to realistic complex structures
4. Based on these findings, propose general directions and specific topics for future research
5. Write a review paper that provides a comprehensive and structured summary of the literature, presents conclusions on the state of the art, and proposes future research directions

In the short term, the outcome of this project will be the review paper mentioned in objective 5. In the long term, it is hoped that this work will help pave the way for the development of a fundamental analysis technique for the mid-frequency range, which will complete the set of tools available to engineers for analyzing structural dynamics across the frequency spectrum.

ACCOMPLISHMENTS AND NEW FINDINGS

In order to prepare the review paper, an extensive literature search was performed on mid-frequency vibration analysis and related topics. Over 250 references were collected, including journal articles, conference papers, doctoral dissertations, and technical reports. The mid-frequency literature was organized into three categories:

- Low- to mid-frequency techniques, such as finite-element-based reduced order modeling methods that aim to reduce the computational costs as frequency *increases* toward the mid-frequency range
- High- to mid-frequency techniques, such as modified formulations of SEA that aim to improve the accuracy as frequency *decreases* toward the mid-frequency range
- Specialized and hybrid techniques that specifically target the mid-frequency range

Following this categorization, the major approaches developed to date were identified.

It was found that the mid-frequency range poses significant challenges in these areas:

- **Connection:** The treatment of a complex structure as an assembly of connected component structures is both a natural starting block and a natural stumbling block in

the mid-frequency range. For instance, different component structures may be best described by qualitatively different models for the same load case: one substructure may have relatively high modal density, the other low modal density. Also, the wave-mode duality of structural vibration is especially important in the mid-frequency range. Incompatibilities between a modal approach and a wave- or energy-based approach must be resolved if one is to consider hybrid methods that employ both types of analysis. Even if a hybrid approach is not taken, each interface between components involves vibration, wave, and energy transmission issues that are not easily resolved. Furthermore, complicated and/or jointed connections require specialized modeling. In terms of problem formulation, the partitioning of the model into appropriate substructures is a crucial step and may not always be obvious or convenient, particularly with respect to specialized mid-frequency analysis methods.

- **Computation:** There is a critical accuracy versus efficiency trade-off in the mid-frequency range. From a low-frequency perspective, refining a finite element mesh, running the finite element analysis, and extracting key results from the analysis can be prohibitively expensive as frequency increases. Therefore, approximate methods must be adopted. From a high-frequency perspective, the situation is reversed. The simplifying assumptions that enable efficient analysis methods in the high-frequency range may not be appropriate for the mid-frequency range, leading to a quantitative and even qualitative breakdown in modeling accuracy as frequency decreases.
- **Prediction:** As the frequency of vibration increases from the low- to mid-frequency range, parameter uncertainties have a greater influence on the response, especially as the wavelength decreases to the scale of random structural variations (e.g., manufacturing tolerances). At some point in the mid-frequency range, a deterministic model represents at best one member in the population of structures with the same nominal design, such that uncertainty in the system must be considered in order to predict the response. Estimated parameters and unmodeled structural complexities provide additional sources of uncertainty. Furthermore, from an engineering perspective, it is important not only to predict the response for a particular design, but also to predict the effect of design changes on that response.

Of these three main areas, it was determined that the primary challenges of the mid-frequency range are related to the first area, the handling of substructure connections. It was also found that the best progress to date has been achieved in the second area—specifically, in improving the computational efficiency of finite-element-based analysis so as to push its range of application higher, into the mid-frequency range.

A major shortcoming of most mid-frequency techniques is that they are either restricted to simple structures, or they cannot be readily applied to realistic complex structures. Therefore, it is clear that more effort needs to be expended on delivering tools that can be

employed by engineers to solve real problems. This does not imply that fundamental research in this area is not important or instructive; rather, basic research must be complemented by and/or guided by applications. In addition, it is important to consider how a method enables not only the analysis but also the design of complex structures.

Based on these findings, the following future research directions are proposed:

- Tailored finite element models and methods
- Physics-based domain decomposition techniques and related reduced order modeling methods
- Smart hybrid methods that can assign automatically assign different modeling techniques to different substructures
- Improved modeling of joints and interfaces
- Applications to realistic complex structures and establishment of benchmarks
- Optimized methods for special classes of structures

Finally, it is suggested that a fundamentally new approach could be developed that takes the best aspects of both FEA and SEA.

For a more comprehensive documentation of this work, a draft of the review paper that resulted from this project is appended to this final report.

PERSONNEL SUPPORTED

The following personnel were supported by and/or associated with this research program:

- Christophe Pierre, Professor
- Matthew P. Castanier, Associate Research Scientist

The personnel listed above are employed by the Department of Mechanical Engineering at The University of Michigan.

PUBLICATIONS

None. However, the result of this work is a review paper that will be submitted for journal publication. For reference, a draft of this paper is appended to this final report.

INTERACTIONS AND TRANSITIONS

The result of this project is a review paper on mid-frequency vibration analysis techniques. This paper will be submitted for journal publication. By publishing this review paper in the archival literature, this work will serve as a valuable reference for engineers and researchers seeking to solve mid-frequency vibration problems. This paper not only summarizes the mid-frequency analysis techniques to date, but it also draws important conclusions on the state of the art and proposes future research directions. By detailing the current challenges in the field and suggesting future topics for investigation, it is hoped that this work will spark new initiatives and breakthroughs in this important area of structural dynamics.

Mid-Frequency Vibration Analysis of Complex Structures: State of the Art and Future Directions

Matthew P. Castanier

Christophe Pierre

Department of Mechanical Engineering

The University of Michigan

Ann Arbor, MI 48109-2125

ABSTRACT

In this work, the literature on mid-frequency vibration analysis of complex structures is reviewed. First, low-frequency and high-frequency techniques that provide a foundation for mid-frequency analysis, or that aim to expand the spectrum of application into the mid-frequency range, are selectively reviewed. Then, specialized and hybrid techniques that specifically target the mid-frequency range are covered. Following the review of existing methods, some general comments on the state of the art are provided. Based on the key issues and challenges that are identified, suggestions for future research are offered.

1 INTRODUCTION

One of the last frontiers in linear structural dynamics lies not beyond the realm of current techniques, but in the middle, surrounded by well-explored regions. The mid-frequency range of vibration is, by definition, bounded from below by the low-frequency range in which finite element analysis (FEA) is well established, and from above by the high-frequency range in which statistical energy analysis (SEA) is popular. In order to understand the defiant juxtaposition of the mid-frequency range, one must examine the limitations of low- and high-frequency techniques.

Starting from a low-frequency perspective, as the frequency of vibration increases, a finite element mesh must be refined in order to capture the shorter wavelengths of vibration. This mesh refinement increases the number of degrees of freedom (DOF) in the finite element model. At some point, the model of a complex structure will become so large that it makes FEA prohibitively expensive. Furthermore, the effects of parameter uncertainties on the structural dynamics [1] become significant as the frequency increases. Regarding the series of modes of a complex structure, Hodges and Woodhouse noted that "the individual modes high up the series become increasingly sensitive to details of the physical structure under investigation, to such an extent that they may be influenced by the deviations from ideal design which inevitably occur in construction" [2]. In addition to these random deviations (e.g., material property variations and manufacturing tolerances), inaccuracies and approximations in the modeling constitute another form of uncertainty in vibration analysis. Therefore, at higher frequencies, it becomes not only more computationally expensive to perform FEA, but it may also be necessary to employ a statistical treatment that drives the analysis requirements beyond the capabilities of the hardware or software.

In contrast, from a high-frequency perspective, the high modal density of a complex structure in this region, combined with the strong influence of uncertainties, make it more appropriate to model the vibration based on the time- and space-averaged response of groups of modes, as is done in SEA. This leads to a smooth estimate of the system response with respect to frequency, which does not rely on a detailed description of the geometry. As the frequency decreases, however, individual resonances first become evident and then become dominant in the actual frequency response of a complex structure. Therefore, at lower frequencies, models with higher fidelity are required to capture key response characteristics.

Clearly, there are frequency limitations for both FEA and SEA. However, there is no established technique analogous to FEA or SEA for the resulting mid-frequency range that resides between their ranges of application. Furthermore, complex structures may have some components that exhibit low-frequency behavior and others that exhibit high-frequency behavior, which poses another class of mid-frequency problems that cannot be treated by established techniques.

As a result of these issues, the mid-frequency range is receiving an increasing amount of interest from researchers. For example, in summarizing the outcome of a recent forum on the current status and future directions of structural dynamics [3], Inman and Ewins identified "mid- to high-frequency modelling" as one of nine key issues for future research [4]. Moreover, they made the following comment on the state of the art:

In assessing how effective the structural dynamics community has been in meeting these expectations, it is clear that current techniques of structural dynamics are very effective for low frequency, linear, deterministic, relatively low-order structures in nice environments. It is equally clear that the current state of practice in structural dynamics is not very effective at treating non-linear, stochastic, mid-frequency, mixed-field problems. [4]

Mid-frequency problems are also receiving attention in industry. For example, mid-frequency vibration problems in the automotive industry were recently noted by Nack [5]. In addition, measurements of car-to-car variability of frequency response functions in this range have been reported [6]. With the emergence of hybrid and electric vehicles that require lighter body structures, mid-frequency vibration problems in the automotive industry could increase significantly in the next decade. Mid-frequency problems have also been examined in other industries, such as the aerospace industry. For example, a study on mid-frequency model development for aircraft fuselages was recently presented [7].

In this paper, the literature on mid-frequency vibration in complex structures is reviewed. In section 2, some background is provided on low- and high-frequency vibration analysis techniques, and the "mid-frequency gap" resulting from limitations of those techniques is discussed from various points of view found in the literature. In section 3, low-to-mid-frequency and high-to-mid-frequency methods are selectively reviewed. Emphasis is given to work that has provided important foundations for mid-frequency methods, as well as efforts to expand the spectrum of low- or high-frequency analysis toward the mid-frequency range. In section 4, specialized and hybrid mid-frequency techniques are covered. Comments on the state of the art and suggestions for future research are provided in sections 5 and 6, respectively. Conclusions are summarized in section 7.

2 BACKGROUND

In this section, some key low- and high-frequency vibration analysis methods are briefly summarized, and their limitations in the mid-frequency range are noted. In addition, the mid-frequency range is considered from various points of view, and a simple definition is established for the purpose of facilitating subsequent discussions.

2.1 Low-Frequency Vibration Analysis

The low-frequency range is characterized by well-spaced modes and distinct resonant peaks in the frequency response function. An illustration is shown in Fig. 1, which is from the book by Ohayon and Soize [8].

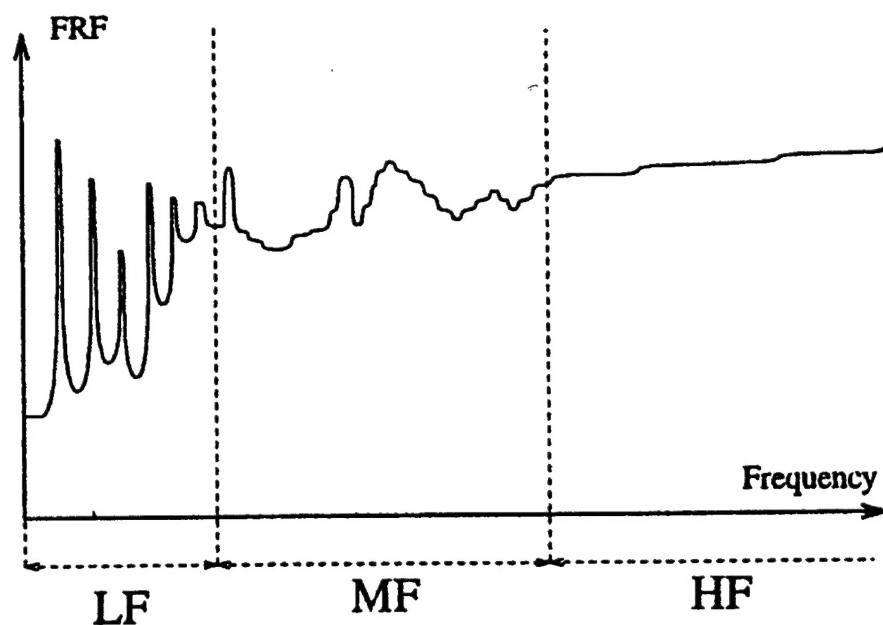


Fig. 1: From [8]: a “qualitative diagram” illustrating structural response in the low-frequency (LF), mid-frequency (MF), and high-frequency (HF) ranges.

In the low-frequency range, the vibration response can be predicted by modal analysis and/or finite element analysis. In fact, FEA in the low-frequency range is considered the gold standard for structural dynamics analysis. However, as the frequency increases, the finite element mesh must be refined in order to capture shorter-wavelength vibration, which can lead to a large number of DOF in a finite element model. Even without pushing the limits of fine meshes, large-scale models such as automotive vehicles can have millions of DOF, making it unwieldy to compute even the first few modes with FEA. Thus, cost is a major limiting factor for using FEA in the mid-frequency range.

Another key issue for the use of FEA in the mid-frequency range is that the wavelengths become sufficiently small that parameter uncertainties have a significant effect on the response. In

a recent paper, Hasselman et al. [9] made this observation on FEA as the frequency reaches the mid-frequency range:

Conventional FEA breaks down in part because the resulting modes do not correlate well with their experimental counterparts. This is due to a combination of product (or test article) variability, relative to nominal design information upon which the FEMs are based, and experimental variability. In addition, and partly because of this variability/uncertainty, FEM meshes have not been pushed to the limits of what modern computer software and hardware are capable of. [9]

Thus, in the mid-frequency range, a deterministic model may not match the actual structural dynamics due to parameter variations. More precisely, above some frequency the deterministic model represents—at best—one member in a population of structures with the same nominal design.

2.2 High-Frequency Vibration Analysis

The high-frequency range is characterized by high modal density, strong modal overlap, and an attendant smoothing effect in the frequency response. This is depicted in Fig. 1. Also, high-frequency structural response is sensitive to uncertainties in the system parameters. The modes, if they can be resolved at all, can vary greatly for a population of structures with the same nominal design. These factors suggest that energy-based techniques and statistical treatments should be used. Therefore, it should not be surprising that a standard high-frequency analysis approach is called statistical energy analysis.

SEA [10–16] is clearly the most popular and established approach to high-frequency vibro-acoustic analysis. In the SEA framework, a structure is divided into a number of subsystems. These subsystems are not necessarily just physical components, but can also be the individual wave fields in the components (e.g., flexural waves versus compression waves). Each subsystem is characterized by the number of modes in a frequency range of interest. Due to uncertainty, all resonant frequencies are considered to have uniform probability of occurring anywhere in the frequency band. Thus, all modes are considered to be resonant modes vibrating with equal energy. This is called “equipartition” of the modal energy. In addition, a diffusive theory of vibrational power flow is employed. The power transmitted from subsystem 1 to subsystem 2 is modeled by the relation [15]:

$$\Pi_{12} = \omega \eta_{12} \left[E_1 - \frac{N_1}{N_2} E_2 \right] = \omega (\eta_{12} E_1 - \eta_{21} E_2) \quad (1)$$

where ω is the center frequency, E_1 and E_2 are the total energies of subsystems 1 and 2, N_1 and N_2 are the number of resonant modes for subsystems 1 and 2, and η_{12} and η_{21} are the *coupling loss factors*. The coupling loss factor is so named because, for instance, the term $\omega \eta_{12} E_1$ represents the power lost by subsystem 1 due to the coupling with subsystem 2. A coupling loss factor is thus analogous to a damping factor.

SEA is a compelling approach in that it provides meaningful predictions, expressed in terms of a small set of parameters and variables, for systems with complex (one might say horribly messy) conditions: high frequency; broadband, spatially distributed excitation; significant parameter variations; etc. In fact, it works especially well when the frequency is sufficiently high, the subsystems are sufficiently complicated, and/or the uncertainty is sufficiently great that the details of the system

are best ignored in favor of a statistical treatment. However, the smooth, averaged response predictions obtained with SEA become less meaningful in the mid-frequency range, as the details of the structure and the substructure coupling become more important, and the individual resonances become evident.

2.3 The Mid-Frequency Gap

As a result of the limitations of established low- and high-frequency techniques, there exists a mid-frequency region with no standard analysis approach. This mid-frequency gap was noted by Cuschieri [17] in a 1987 paper:

Therefore SEA and FEA, while both are extremely useful in their respective frequency regions, can leave an empty gap in the mid-frequency range, where the modal density is not high enough for frequency averaging to give reliable results, but where a number of modes are present which can make the FEA unwieldy. [17]

In a 1998 paper, Hasselman et al. [18] took a slightly different view of the mid-frequency gap, citing the difference between deterministic and statistical approaches:

Ultimately a frequency regime is reached where there may not be a single resonant mode in a given frequency band, and the "average" response predicted by an SEA model is meaningless. Unfortunately, the frequency range associated with sparse modes, or low modal density, in statistical energy analysis usually coincides with the frequency regime where deterministic analysis of the total system becomes impractical. A prediction model is required for this intermediate frequency range, to provide a bridge from low-frequency (deterministic) models to high-frequency (statistical) models. [18]

A third view is that the mid-frequency gap is due to a mixture of short and long wavelength behavior, as summarized by Langley and Bremner [19] in 1999:

The conditions required for the successful application of SEA have been the subject of considerable previous work..., and it is generally recognized that, in addition to other conditions, each subsystem must ideally contain a number of resonant modes over the analysis band of interest. One implication of this condition is that the wavelength of the subsystem deformation must be of the same order as, or less than, the dimensions of the subsystem. In some cases this requirement may be only partially met: for example, in-plane waves in a plate are generally of much longer wavelength than bending waves, so that while the bending motion might meet the SEA requirement, the in-plane motion might not. While such difficulties can, in some cases, be overcome by employing problem specific modeling techniques within SEA..., it is true to say that a general approach is lacking. The problem of the existence of both short and long wavelength deformations within a structure is likely to become more severe with *decreasing* frequency, in the sense that all wavelengths will be sufficiently short at a sufficiently high frequency. This creates a difficult mid-frequency zone between low-frequency finite element modeling and high-frequency SEA modeling. [19]

All of these views are valid, but it should be noted that not all of the above criteria need to be met in order to define the mid-frequency range. In fact, if any of these criteria are met, one has found a mid-frequency problem.

In this work, a pragmatic engineering point of view is adopted for defining the mid-frequency range, such that all of the above perspectives are included. The mid-frequency range for any structural dynamics problem is considered to be an intermediate frequency range for which there is no readily available method, such as FEA or SEA, to handle the analysis of the full complex structure in a systematic fashion with acceptable efficiency and accuracy relative to a lower or higher frequency region. This definition, of course, depends not only on the details of the system but also on the needs of the analyst. Furthermore, this definition implies that there is an available method that can provide acceptable results in a lower or higher frequency range.

3 EXCURSIONS INTO THE MID-FREQUENCY RANGE

In this section, low- and high-frequency analysis techniques that expand the range of applicability toward the mid-frequency range are considered. Special attention is given to methods that have been used as a basis for mid-frequency analysis, as well as to extensions that have been proposed to overcome the challenges associated with the mid-frequency range. Several of these approaches have been specifically presented by the authors as applicable across the low- to mid-frequency or the high- to mid-frequency spectrum.

3.1 Pushing Low-Frequency Analysis Higher

As a result of the high cost of FEA for large-scale models or fine meshes, substructuring techniques have been developed. These techniques make manageable the necessary finite element analysis through a divide-and-conquer approach. In addition, they enable the generation of reduced order models. Therefore, they provide a foundation for increasing the scope of the analysis into the mid-frequency range.

3.1.1 Component Mode Synthesis

A popular approach for generating reduced order models systematically from finite element models is component mode synthesis [20–30]. In component mode synthesis (CMS), a complex structure is partitioned into several component structures or substructures. (In this study, the terms “components,” “component structures,” and “substructures” are used interchangeably.) The vibration of the full complex structure is represented by a set of modes selected for each component structure, plus a set of vectors that are used to couple the components at their interfaces. The CMS approach was introduced by Hurty [20] in 1965, who suggested using fixed-interface modes for the components. In 1968, Craig and Bampton [21] introduced an improved fixed-interface CMS method. Goldman [22] showed that free-interface modes could be used instead of fixed-interface modes. MacNeal [23] introduced a hybrid method that allowed a combination of fixed and free interface DOF, and he also presented the residual flexibility method that accounts for the flexibility of the discarded modes. Rubin [24] extended MacNeal’s residual flexibility method by also including residual inertial and dissipative terms. Hintz [25] also provided improvements to and observations on previous CMS methods. Hale and Meirovitch [27] showed that admissible functions could be used to model the substructures, which yields a more general method of CMS. An overview of the CMS approach was provided by Craig [28]. There have been several reviews of the CMS literature, including the papers by Craig [29] and Seshu [30].

The CMS approach has several advantages. Through substructuring, the model sizes for the required finite element calculations are reduced from the order of system DOF to the order of component DOF. Another clear computational advantage is that the final system model can be significantly smaller than the original finite element model while retaining excellent accuracy for a frequency range of interest. Furthermore, since the component models are separated, the components can be designed and re-designed separately before being assembled in the dynamic analysis.

The most popular CMS method seems to be the Craig-Bampton method [21]. It is an exceptionally efficient and stable method, and it has been implemented in many commercial software programs. For example, it forms the basis of the superelement capability in the finite element

code NASTRAN, and it is employed in the flexible element available in the mechanical system simulation code ADAMS. In the Craig-Bampton method, fixed-interface component modes are complemented by the so-called constraint modes. A constraint mode is the static deflection induced in a substructure by a unit displacement at one interface DOF, with all other interface DOF held fixed. Thus, there is one constraint mode for each interface DOF in the finite element model. The components are then assembled into a system model by enforcing displacement compatibility at the interfaces. For simplicity, consider the case of only two substructures. The system matrices are of the form [21]:

$$\mathbf{M}_{\text{CMS}} = \begin{bmatrix} \mathbf{m}_1^{NN} & \mathbf{0} & \mathbf{M}_1^{NB} \\ \mathbf{0} & \mathbf{m}_2^{NN} & \mathbf{M}_2^{NB} \\ \mathbf{M}_1^{BN} & \mathbf{M}_2^{BN} & \mathbf{M}^{BB} \end{bmatrix}, \quad \mathbf{K}_{\text{CMS}} = \begin{bmatrix} \mathbf{k}_1^{NN} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{k}_2^{NN} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}^{BB} \end{bmatrix} \quad (2)$$

where superscript N denotes normal-mode coordinates (component-mode DOF), superscript B denotes boundary coordinates (constraint-mode DOF), and the subscript refers to the substructure number. Note that \mathbf{m} and \mathbf{k} are matrix partitions that have been reduced in both dimensions by the modal truncation, while \mathbf{M} and \mathbf{K} are matrix partitions that have at least one dimension equal to the number of interface DOF, which has not been reduced from the finite element representation. Strategies for the reduction of these interface DOF are discussed in section 3.1.2.

In 1997, Shyu et al. [31] introduced a variation on traditional CMS that combines the constraint mode approach with dynamic compensation. Instead of using static constraint modes, quasi-static constraint modes are calculated about a centering frequency for the frequency range of interest. This captures inertial effects of the truncated modes. The authors noted that this approach is "ideally suited for mid-band frequency analysis in which both high-frequency and low-frequency modes may be omitted." In a subsequent paper [32], criteria were presented for selecting the centering frequency and the quasi-static mode sets for both low-frequency and mid-frequency response calculations.

3.1.2 Secondary and Multi-Level Reduction Methods

A drawback of fixed-interface CMS methods is that the finite element DOF for all interfaces between component structures are retained in the system model as constraint mode DOF. Thus, while the component modes can be selected for a certain frequency range in order to reduce the model size, the interface DOF are not reduced. In some cases, the interface DOF can dominate the size of the model.

In a 1977 NASA report, Craig and Chang [33] considered several methods for reducing the interface DOF, which they referred to as junction coordinates. One method involved constraining the component mode coordinates and allowing motion of junction coordinates to yield an eigenvalue problem of the form:

$$\mathbf{K}^{BB}\psi = \lambda\mathbf{M}^{BB}\psi \quad (3)$$

where \mathbf{K}^{BB} and \mathbf{M}^{BB} are the constraint-mode partitions from Eq. (2), λ is an eigenvalue, and ψ is an eigenvector. Craig and Chang noted that this corresponds to using Guyan reduction [34] to reduce out the interior coordinates of the components.

These junction modes can then be selected and used to reduce the interface DOF, as in a traditional modal analysis. Again, considering the simplest case of two substructures, this yields

matrices of the form:

$$\mathbf{M}_{\text{red}} = \begin{bmatrix} \mathbf{m}_1^{NN} & 0 & \mathbf{M}_1^{NB}\Psi \\ 0 & \mathbf{m}_2^{NN} & \mathbf{M}_2^{NB}\Psi \\ \Psi^T \mathbf{M}_1^{BN} & \Psi^T \mathbf{M}_2^{BN} & \Psi^T \mathbf{M}^{BB} \Psi \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1^{NN} & 0 & \mathbf{m}_1^{NJ} \\ 0 & \mathbf{m}_2^{NN} & \mathbf{m}_2^{NJ} \\ \mathbf{m}_1^{JN} & \mathbf{m}_2^{JN} & \mathbf{m}^{JJ} \end{bmatrix} \quad (4)$$

$$\mathbf{K}_{\text{red}} = \begin{bmatrix} \mathbf{k}_1^{NN} & 0 & 0 \\ 0 & \mathbf{k}_2^{NN} & 0 \\ 0 & 0 & \Psi^T \mathbf{K}^{BB} \Psi \end{bmatrix} = \begin{bmatrix} \mathbf{k}_1^{NN} & 0 & 0 \\ 0 & \mathbf{k}_2^{NN} & 0 \\ 0 & 0 & \mathbf{k}^{JJ} \end{bmatrix} \quad (5)$$

where J denotes the junction mode coordinates. Note that all partitions have now been reduced by modal truncation. This approach is a straightforward way to generate very small system models of a complex structure for a frequency range of interest.

Unfortunately, Craig and Chang did not publish this work in a conference proceedings or a journal, so it has remained a little known contribution. There have been many similar efforts since then, including several by authors who were not aware of Craig and Chang's work. In fact, Brahmi et al. [35] and Castanier et al. [36] derived the junction modes independently. Castanier et al. called them characteristic constraint (CC) modes, since they are eigenvector-based linear combinations of the constraint modes. They took the analysis further by showing how the CC modes capture the primary interface motion and thus provide a convenient framework for power flow analysis. They investigated CC-mode-based analysis of vibration and power flow in several subsequent studies [37–41]. This work will be discussed in section 3.1.3.

Other efforts in the area of interface reduction and improved substructuring methods include the following. Bourquin and d'Hennezel [42, 43] introduced a fixed-interface CMS method "based on a non-conventional choice of constraint modes tied to the normal modes of the Poincaré-Steklov operator associated with the interface between the substructures" [42]. They called these alternative constraint modes the "coupling modes," and they proposed algorithms for handling the various numerical computations needed to find these coupling modes [43].

Bouhaddi and co-workers [35, 44–50] have proposed methods for improved dynamic condensation and Guyan reduction, two-level dynamic condensation, and interface reduction. For the interface DOF, they investigated reduction of junction coordinates not only for a synthesized system model [35, 48, 49], but also for the individual substructure models before assembly [46]. The latter approach requires analysis only for smaller sub-problems, and it is well suited for implementation in design optimization.

Balmès [51–54] considered several interface reduction methods as well as more general issues related to finding optimal bases for reduced models. He calculated the junction modes based on a singular value decomposition of the matrix of constraint modes [51]. (It seems that Balmès also found the junction modes independently, since the Craig and Chang report was not referenced in [51]. However, in subsequent papers [52, 54], Balmès cited Craig and Chang for the junction modes, and he noted that his work in [51] demonstrated that the junction modes correspond to an "optimal selection of generalized constraint modes" [52].) Balmès also proposed using arbitrary interface models to find a basis for representing the interface motion. For example, he suggested making a local finite element model by retaining the elements connected to the finite element nodes of the interface, and then finding the interface modes by performing a modal analysis of the local model with free boundary conditions [52].

Ohayon et al. [55] also proposed a singular value decomposition to find reduced models of substructure coupling. They employed both fixed- and free-interface modes, and performed singular

value reduction on frequency-independent Lagrange multiplier terms. Rixen et al. [56] introduced a hybrid procedure for the dynamic analysis of general substructure problems using Lagrange multipliers, including the case of non-conforming finite element meshes at the interface. Rixen [57] later proposed interface reduction via "force modes," or modes of the coupling forces between substructures. These force modes are based on the interface flexibility matrix, and they are conceptually similar to junction modes.

Recently, Aoyama and Yagawa [58, 59] introduced an alternative method for finding interface modes by performing a modal analysis for each pair of connected components. For any two connected components, the common interface is included in the modal analysis while the interface DOF shared with other components are held fixed. This effectively yields interface modes for the interface between the two components. Aoyama and Yagawa showed large order reductions using this method compared to a traditional CMS method.

Another strategy for handling large-scale models is multi-level substructuring [60–63]. In this approach, the component structures are partitioned again into sets of sub-components to reduce the computational costs associated with calculating component modes and constraint modes. The sub-components are assembled back into their parent components, and the components are then assembled into the system model. Of course, this may be applied recursively for many levels of substructures. Recent applications of multi-level substructuring include the work of Mourelatos [64] and Tan et al. [38].

A notable initiative in this area is the adaptive/automated multi-level substructuring (AMLS) approach of Bennighof [65–71]. In AMLS, a finite element model is substructured recursively such that each of the substructures at the lowest level "consists of a small number of finite elements" [67]. The Craig-Bampton method of CMS [21] is used to assemble the child substructures into their parent substructures. The overall computational costs are relatively low compared to conventional FEA of the full structure. Furthermore a frequency window technique for AMLS has been presented [67] to enhance the accuracy and efficiency of the approach, such that the cost of AMLS for a frequency band is on the order of that of FEA for a single frequency. To date, no other implementation of multi-level substructuring seems to be as aggressive and systematic as the AMLS approach.

3.1.3 Power Flow and Uncertainty Analysis

Since component mode synthesis provides a system model in terms of coupled substructures, it seems natural to use it as a lower-frequency alternative to SEA for power flow and uncertainty analysis. In fact, Lyon and DeJong compared the coupling loss factor for power flow in SEA to the interface model in component mode synthesis [15]:

The coupling loss factor or its various equivalent expressions is a measure of inter-modal forces at the system junction, averaged over frequency and over the modes of the interacting systems. Sometimes this calculation may be carried out directly. Such a calculation is also the basis for the method of component mode synthesis as used in finite element analyses. [15]

However, it is only recently that using CMS as a basis for power flow calculations has received significant attention.

In 2000, Mace and Shorter [72] presented energy flow models using both global and local finite element analysis. For the local analysis, they employed the Craig-Bampton method of CMS. In doing so, they noted that a "CMS model is particularly well suited for postprocessing into an energy flow model, since the global degrees of freedom of the structure are easily partitioned in subsystem degrees of freedom." They showed numerical results for a three-plate system, and found the response to have resonances due to subsystem mode-pair interactions that were not captured by the SEA results. They found the CMS-based approach to be computationally efficient. In 2001, Mace and Shorter [73] used CMS as a basis for performing mid-frequency analysis of built-up structures. They used perturbation to relate small changes in the component modal properties to resultant changes in the global modal properties. They then performed Monte Carlo simulation to estimate the statistics of the frequency response function at relatively low computational cost.

Significant progress in CMS-based low- to mid-frequency vibration analysis has been made in recent work by Pierre and co-workers [36–40, 74, 75]. An investigation into a general framework for predicting the power flow between coupled component structures with uncertain parameters was performed by Tan et al. [74]. In this study, the power flow between two beams was modeled with CMS by computing the power transmitted through the constraint mode degrees of freedom. The ensemble-averaged power flow was estimated by expanding the modal responses in terms of locally linear interpolation functions in the random parameter space. Good agreement was found with wave-based approximations from the literature. However, it was seen that a large number of interface DOF could render a power flow analysis with traditional CMS rather inefficient.

In order to reduce the size of a CMS model and provide a convenient basis for power flow analysis, Castanier et al. [36] found that a secondary modal analysis could be performed on the constraint-mode coordinates in order to retrieve characteristic constraint (CC) modes. The CC modes are thus a linear combination of the constraint modes that capture the primary motion in the interface region with relatively few vectors. They can be selected for a certain frequency range and used to reduce the system model, as in a traditional modal analysis. (The topic of interface reduction was covered in section 3.1.2.)

For illustration, consider the cantilever plate shown in Fig. 2. The plate is partitioned into two components, called plate 1 and plate 2. A constraint mode for plate 2 is shown in Fig. 3. Recall that a constraint mode is the static deformation due to a unit displacement of a single interface DOF, with all other interface DOF held fixed. It can be seen that this yields a rather localized deformation shape. In contrast, consider the 6th and 7th CC modes for plate 2 shown in Fig. 4. It can be seen that the CC modes capture more natural, wavy motion of the interface. Furthermore, since the complementary sets of modes in the system model are fixed-interface component modes, the interface motion—and thus the power flow between substructures—is completely described by the CC modes. Therefore, the CC modes provide a basis for reduced order models of power flow. This is illustrated in Fig. 5, which shows the 6th and 7th CC modes again for the coupled system, which is the full plate. The motion in the interface region, with participation from both component structures, is clearly seen.

Characteristic-mode-based models of power flow were developed for both deterministic and statistical analyses in several papers by Tan et al. [37–40]. It was found that such models provide a framework for efficient low- to mid-frequency vibration analysis of complex structures. Furthermore, the approach is general, and it can be applied to large-scale engineering structures such as military or automotive vehicles [38, 40]. In a recent investigation [75], the method was implemented in NASTRAN using Direct Matrix Abstraction Program (DMAP) routines, in order to

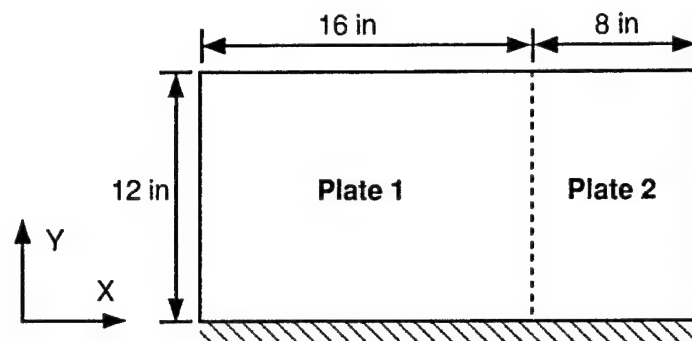


Fig. 2: A cantilever plate partitioned into two components.

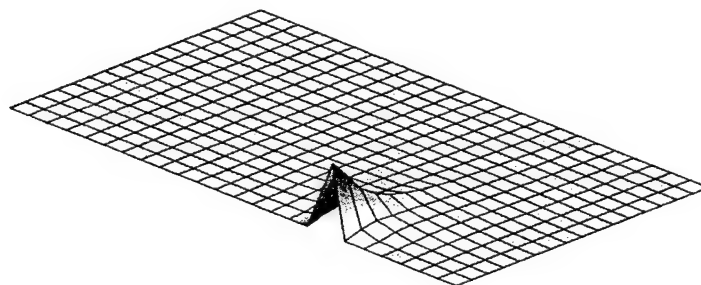


Fig. 3: A constraint mode for plate 2.

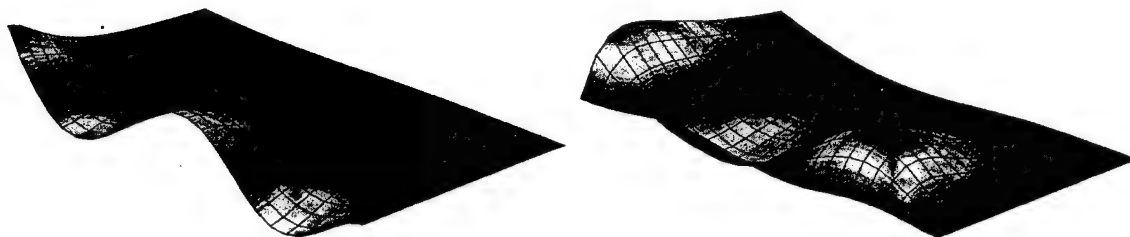


Fig. 4: The 6th and 7th CC modes (only plate 2 shown).

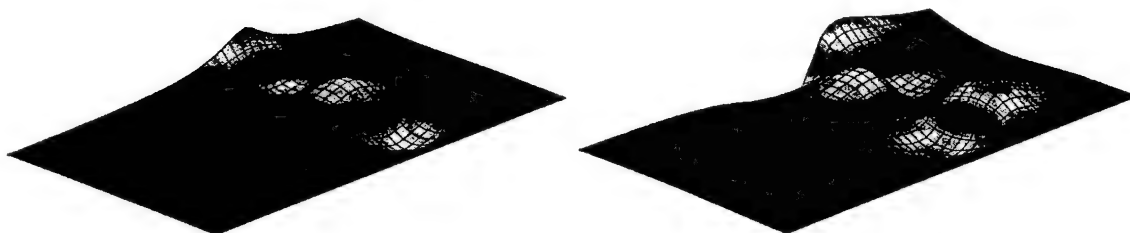


Fig. 5: The 6th and 7th CC modes (full plate shown).

handle a vehicle finite element model with 1.5 million DOF. For the range 0–200 Hz, a reduced order model with only 2124 DOF was found to predict the vibration response and power flow in the structure with excellent accuracy relative to the full finite element model.

3.2 Pulling High-Frequency Analysis Lower

3.2.1 SEA Enhancements, Extensions, and Alternative Formulations

The underlying assumptions and resultant limitations of SEA have been examined by many researchers, including Woodhouse [76], Hodges and Woodhouse [2], Keane and Price [77], Langley [78, 79], Fahy [80], and Mace [81]. These papers have provided great insight into the basic foundation of SEA. In addition, Dowell and Kubota [82] found the SEA results by investigating the asymptotic limit of classical modal analysis, an approach they called Asymptotic Modal Analysis [82–88]. The state of the art of SEA and alternative high-frequency analysis methods was recently reviewed by Sestieri [89].

Among other limiting assumptions, equipartition of modal energy and weak coupling between subsystems have been cited as key conditions for SEA. In fact, Langley [78] presented a general formulation of SEA and derived the standard SEA equations under the assumption of weak coupling. This is an important restriction, as it will be seen in subsequent sections that the modeling of the interface between substructures is a critical aspect of mid-frequency techniques. In 1992, Langley [90] examined the assumption of equipartition of vibration energy among the resonant modes, which is equivalent to assuming diffuse wavefields in each structural component. He noted that the diffuse wavefield assumption of SEA can lead to poor results due to the filtering effect of interfaces between components. Langley relaxed this assumption by modeling the vibrational wave intensity with a Fourier series, an approach called wave intensity analysis [90, 91]. By taking a single Fourier term, the conventional SEA formulation is found.

In 1987, Keane and Price [77] carefully considered the assumptions of SEA, with emphasis on the strength of coupling between subsystems. Using an example of a pair of one-dimensional, point-spring-coupled subsystems, they investigated the effect of relaxing certain assumptions. For structural systems with a few widely-spaced modes in the frequency range of interest, they suggested that rather than assuming that all resonant frequencies are equally likely to fall anywhere in the frequency range of interest, it might be better to allow a sequence of uniform probability density functions (PDFs) about the individual natural frequencies predicted by deterministic methods. They noted, however, that this would complicate the statistical analysis significantly. In a similar vein, Keane and Price later examined the application of SEA to periodic structures [92]. They modeled the passband-stopband behavior of such systems by implementing a PDF for the natural frequencies that was greater in passband regions and smaller in stopband regions. Building on this work, Pierre et al. [93] related estimates of the variations in subsystem parameters to the distributions of subsystem natural frequencies. Their so-called parameter-based statistical energy method (PSEM) thus captured resonant behavior in the ensemble-averaged power flow in the mid-frequency region. However, this treatment was limited to simple one-dimensional systems and was not easy to generalize.

There have been many other efforts toward improving SEA estimates of coupling loss factors and power flow. For example, Sablik et al. [94] used discretized modal densities to account for structural resonances in a beam network. Fredö [95] combined FEA with an SEA-like (SEAL)

energy flow balance to find the power flow between coupled plates. He derived "energy flow coefficients" that were similar to coupling loss factors, but they were related to the deterministic case, captured resonances, and could become negative at certain frequencies. Simmons [96] used finite element models to calculate coupling loss factors and power flow through plate junctions. Maxit and Guyader [97, 98] have also presented a method for using finite element models to determine SEA coupling loss factors.

Yan et al. [99] implemented this type of combined FEA/SEA approach in commercial software. They used NASTRAN to calculate impedance matrices at every driving frequency, and post-processed them to obtain coupling loss factors. These coupling loss factors were then used in AUTOSEA, a commercial SEA code, to find the system response in the mid-frequency range.

Zhang and Sainsbury [100] introduced an approach called the energy flow method (EFM), which is based on SEA but also uses FEA for connections between strongly coupled subsystems in order to improve on the SEA results in the mid-frequency range. In particular, a method that was presented by Guyader et al. [101, 102] for determining energy flow in coupled plates is employed for handling strong couplings. Guyader's method involves the calculation of energy influence coefficients (EICs) to relate the energy values for connected substructures. The EICs are based on the modes of the coupled system, and they also depend on the excitation. In the energy flow method, it is assumed that many of the subsystem couplings are weak, such that the SEA coupling loss factors are sufficient to describe the power flow for these junctions. For junctions that feature strong coupling, the EICs are first calculated using FEA, and then the EICs are used to determine coupling loss factors. Therefore, the final SEA equations feature conventional coupling loss factors for weakly coupled subsystems and FEA-based coupling loss factors for strongly coupled subsystems. Although there was no restriction on the weakly coupled subsystems, the method was limited to structures with only one pair of strongly coupled subsystems.

3.2.2 Vibrational Conductivity, Power Flow, and Energy FEA

In SEA, a uniform energy, based on the total energy of the resonant modes, is assumed for each subsystem. The power flow between coupled subsystems is computed based on the difference in subsystem energies, as shown in Eq. (1). Lyon has pointed out that, in the SEA system model, the resonant modes are considered to be energy "containers" [103]. If the basic energy container is reduced from a subsystem to a finite element, then the energy density is allowed to vary in each substructure. In the limit of small elements, one retrieves a partial differential equation in which the flow of vibration energy is proportional to the energy gradient. Thus, the flow of mechanical energy is modeled as the flow of thermal energy, yielding a true heat transfer approach to power flow.

The origins of this approach can be found in the Russian literature beginning in the late 1970s [104, 105]. The first treatment in the English language archival literature seems to be that of Belyaev and Palmov [106], who called it the vibrational conductivity approach. Belyaev reported further developments in later papers [107, 108]. The technique was popularized in 1989 by the finite element formulation of Nefske and Sung [109]. They noted that the governing equation was analogous to the finite element equation for heat conduction and thus could be solved by existing finite element software. They applied this finite element analysis to beams. Due to the influential work of Nefske and Sung, the approach is often called power flow finite element analysis (PFMEA) or energy finite element analysis (EFEA). It is important to realize that, unlike traditional FEA for

high-frequency vibration, EFEA does not require a fine mesh. This was explained by Sestieri in a recent review of high-frequency methods [89]:

...the heat equation is a parabolic equation describing a diffusion phenomenon and admits solutions exponentially decaying from the source, without oscillations. On the contrary, the wave equation describes a propagation phenomenon and has oscillating solutions in space, whose wavenumbers increase in direct proportion to the exciting frequency. This difference permits [one] to solve the parabolic equation with a coarse mesh that is usually frequency independent while the wave equation requires a mesh that becomes more and more demanding as the frequency increase[s] and the space passes from one to two and three-dimensions. [89]

The low-cost finite element formulation for the system, along with the spatially varying energy metric for the subsystems, make this approach appealing. In addition, power flow between connected substructures can be handled by calculating *power transfer coefficients*, which are analogous to the coupling loss factors used in SEA.

Key contributions and improvements to EFEA were provided by Bernhard and co-workers [110–113]. Among other contributions, they made important observations on the averaging performed on the energy variable, and they extended the approach to two-dimensional structures in applying it to plates. Palmer et al. [114, 115], Stiehl [116], and Moens et al. [117] applied EFEA to systems of connected beams. Moens et al. also compared the numerical results to experimental results, and they concluded that EFEA is a promising technique for the mid- to high-frequency range.

Langley [118] provided a more general derivation and application of the vibrational conductivity equation for two-dimensional structural components, and he examined fundamental assumptions and limitations of the approach. Carcaterra and Sestieri [119] also gave careful consideration to the use of the thermal analogy of energy flow in structures. They noted that different authors had employed different averaged energies, and thus power flow methods do not necessarily lead to the same heat conduction equation. They also warned that “it is dangerous to use the thermal analogy as an almost exact approach to describe the time-averaged energy density, especially for complex structures.” They later tried to overcome the limitations of the thermal analogy with their “envelope energy” models [120, 121]. In 1999, Xing and Price [122] also noted the general lack of similarity between mechanical and thermal energy concepts, and they introduced an alternative power flow analysis based on the governing equations of continuum mechanics. They showed that “the governing equation of energy flow is a first-order partial differential equation which does not directly correspond to the equation describing the flow of thermal energy in a heat-conduction problem.”

However, these warnings are related to the modeling of the energy distribution within a substructure. Given that the energy is modeled as a locally varying quantity rather than assigning an averaged value, it is possible that EFEA could provide an improvement relative to SEA when applied to the propagation of mid-frequency vibration through complex structures. As a case in point, Vlahopoulos et al. [123] performed a numerical implementation and validation of EFEA for models of marine vessels, and they found good results relative to SEA. They also cited some key advantages of EFEA:

The EFEA formulation offers advantages in three main areas: the model generation is

based on actual geometry, thus uncertainties in defining subsystems and their connections are eliminated; the results can be displayed over the entire system; and spatial variation can be assigned to the design variables when studying alternative configurations for performance improvements. [123]

Building from this work, as well as from a study on calculating power transfer coefficients [124], Vlahopoulos and Zhao have developed a hybrid FEA/EFEA mid-frequency technique [125–127]. This hybrid technique will be covered in section 4.6. Finally, it is noted that recent developments and applications of EFEA have been presented by Bernhard and co-workers [128–130].

3.2.3 Local Energy Flow Analysis

Beginning in the mid-1990s, Jezequel, Le Bot, and co-workers presented alternative formulations to EFEA for modeling the local variation of high- to mid-frequency vibration energy. In 1996, Lase et al. [131] introduced the general energy method (GEM). To develop this method, they considered both the total energy density (sum of kinetic and potential energy densities) and the Lagrangian energy density (difference between kinetic and potential energy densities). The time-averaged total energy density has a hyperbolic spatial distribution. In contrast, the time-averaged Lagrangian energy density has a sinusoidal spatial distribution, and it vanishes for high frequencies or infinite structures. The authors then derived two types of energy flows. The first is the active energy flow, which is related to the total energy and describes the propagation of energy in the system. The second is the reactive energy flow, which is related to the Lagrangian energy and characterizes the modal behavior of the structure. They formulated both active and reactive energy flow balances, with the active energy flow balance being equivalent to that of EFEA. They then applied the general energy method to the vibration of bars and beams. For the case of bars, they found two second-order differential equations governing the spatial distribution of energy. For the case of beams, they retrieved eight second-order differential equations. Although the results exactly matched those of the traditional displacement solution, the general energy method clearly becomes computationally intensive even for simple one-dimensional systems.

To address this problem, the authors performed spatial averaging over a wavelength. With spatial and time averaging, the Lagrangian energy density and reactive power flow were found to be zero. The total energy density was unchanged by spatial averaging, however, leaving the active energy flow balance as the governing relation. They called this approximation the simplified energy method (SEM), and the resulting equations of local energy variation were of the heat conduction type and equivalent to those of EFEA [109, 110]. In subsequent papers, the simplified energy approach was applied to energy flow in plates [132] and in one-dimensional systems with multiple propagating wave types [133].

In an attempt to improve upon the heat conduction approximation, Le Bot and Jezequel recently introduced a local energy flow approach [134–136]. This is a wave-based approach, in which it is assumed that the energy density is a sum of the energy quantities of each propagative field. In addition, the Huygens principle is used, so that the energy is assumed to be a superposition of the direct field and the reverberant field. The direct field is created by primary (actual) sources in the domain. The reverberant field is considered to be created by secondary (fictive) sources located at the boundary of the domain. These wave fields and sources are illustrated in Fig. 6. Based on this approach, the local energy results are found to be “a direct prediction of the mean values of

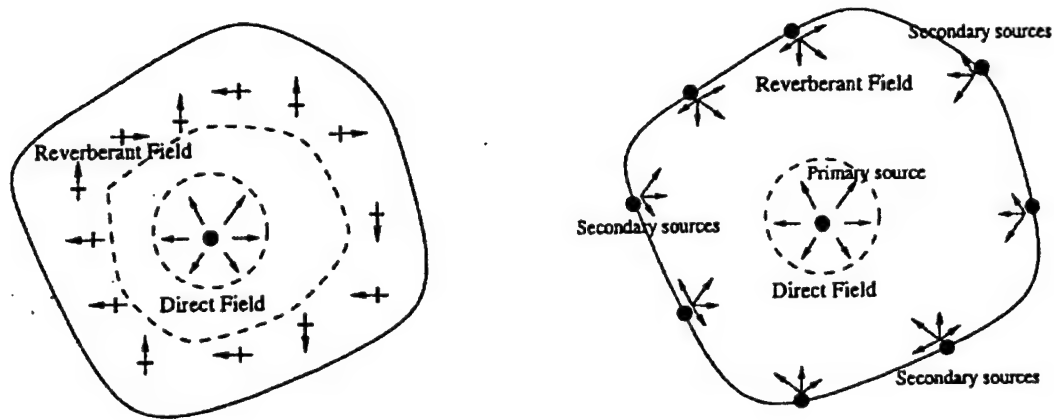


Fig. 6: From [135]: illustration of the wave fields (left) and the direct and secondary sources (right) assumed in the local energy flow approach.

expected dynamical levels” [135]. The local energy flow approach has been applied to coupled beams [135] and coupled plates [134, 136]. One interesting result is that this approach can predict the stopband-passband behavior for a periodic system [135].

The assumption of primary and secondary sources has also been used in the solution of a random boundary element formulation for mid- to high-frequency vibration presented by Viktorovitch et al. [137, 138]. In this method, classical dynamic integral equations (e.g., the Green kernel) are modified by introducing random geometric parameters to model the effects of uncertainties as frequency increases. These equations are also multiplied by “well-chosen” kinematic variables associated with the boundary. The unknowns are the second order stochastic moments of the kinematic variables. Results have been presented for one-dimensional [137] and multi-dimensional [138] structures, including connected substructures. For assemblies of substructures, it is also assumed that a primary source for one substructure acts as a primary source for a connected substructure at the interface. The authors have demonstrated that their approach can capture low-frequency modal behavior as well as smooth, averaged high-frequency response. They have also shown that it can handle connected substructures with large differences in their modal densities. Therefore, this method seems more appropriate for mid-frequency modeling than the related local energy flow approach. However, given the integral formulation used as a basis, it remains to be seen whether this method can be generalized for arbitrary complex structures.

3.2.4 Structural Path Analysis

Girard and Defosse [139] employed the “frequency smoothing effect” of a structure’s driving point mobility or flexibility as frequency increases in order to derive system frequency response functions in the mid- to high-frequency range. They noted that if the asymptotic (smoothed) property is used, it causes problems for a finite-element-like assembly of the stiffness matrix for a structure. Therefore, they developed an alternative approach, based on the inverse of the system stiffness matrix. It was shown that, after eliminating some small terms, the connection between any two nodes can be written as a sum of the contributions of all the “structural paths” connecting those points,

either direct or indirect. It was argued that for high frequencies, only the shortest structural path needs to be retained. For lower frequencies, longer structural paths must be included, but they can still be truncated. This procedure holds for a substructuring approach, but it seems to be limited to point couplings between substructures.

The authors later extended this work to handle beam trusses as well as lumped masses and springs [140]. In this work, it seems that the number of structural paths can become daunting. For one pair of nodes in a 24-beam truss used as an example, the number of "topological paths" is 131, but for just two wave types (longitudinal and flexural) the number of structural paths is 73,000. The authors proposed some ways to handle the large number of paths.

This method of frequency smoothing plus structural path analysis yields the mean frequency response curves for a structure—smooth curves between resonant and anti-resonant levels, similar to SEA. In 1997, Girard et al. [141] summarized the approach and cited two of its main advantages: (1) the response is given as vectors (displacements, velocities, accelerations) rather than scalar energy results, and (2) the path analysis shows how vibration is transmitted through the structure from the excitation source to response points. In addition, they showed that the smoothed FRFs of substructures from analysis or from experimental results can be used to assemble the smoothed system FRF. However, they concluded that further work was needed to improve "the accuracy of FRF substructuring in the medium frequency domain where classical methods generally lead to severe discrepancies." Therefore, while this is clever work, it seems rather limited in terms of mid-frequency applications.

4 SPECIALIZED AND HYBRID TECHNIQUES

In this section, techniques that specifically target the mid-frequency range are covered. These methods are notable in that they employ hybrid techniques and/or new approaches to the modeling and analysis of mid-frequency vibration in complex structures.

4.1 Fuzzy Structure Theory and Mid-Frequency Band Modeling

4.1.1 Fuzzy Structure Theory

Fuzzy Structure Theory was introduced by Soize et al. [142] in 1986 as a way to handle the mid-frequency dynamic analysis of systems with structural complexity. Since then, further developments have been presented by Soize [143, 144] and by Soize and Bjaoui [145]. Also of note is the work by Pierce et al. [146] and Sparrow and co-workers [147–149]. Fuzzy structure analysis has been summarized in a tutorial paper by Ruckman and Feit [150], in a brief paper by Soize [151], and in chapter 15 of the book by Ohayon and Soize [8].

The basic approach is to treat a structure as a combination of a primary structure and a number of secondary structures and attachments. The primary structure is called the *master structure*, and it is the part of the structure that can be modeled deterministically using conventional techniques such as finite element analysis. The complement to the master structure consists of substructures having complexities that cannot be easily modeled and/or properties that are not precisely known. For this reason, they are referred to as *fuzzy substructures*. The entire system, consisting of the master structure plus fuzzy substructures, is called a *fuzzy structure*. An illustration of a fuzzy structure is shown in Fig. 7.

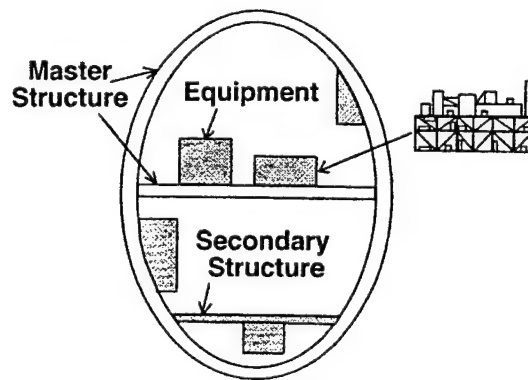


Fig. 7: From [8]: a fuzzy structure consists of a master structure (unshaded) and fuzzy substructures (shaded).

The key concept in fuzzy structure analysis is that fuzzy substructures act like vibration absorbers due to high modal density above their first resonant frequency, and the corresponding transfer of energy from the master structure to the fuzzy substructures acts like added damping for the overall structural response. Ohayon and Soize [8] referred to this effect as “an ‘apparent strong damping’ in the master structure” for the mid-frequency range, as shown in Fig. 8.

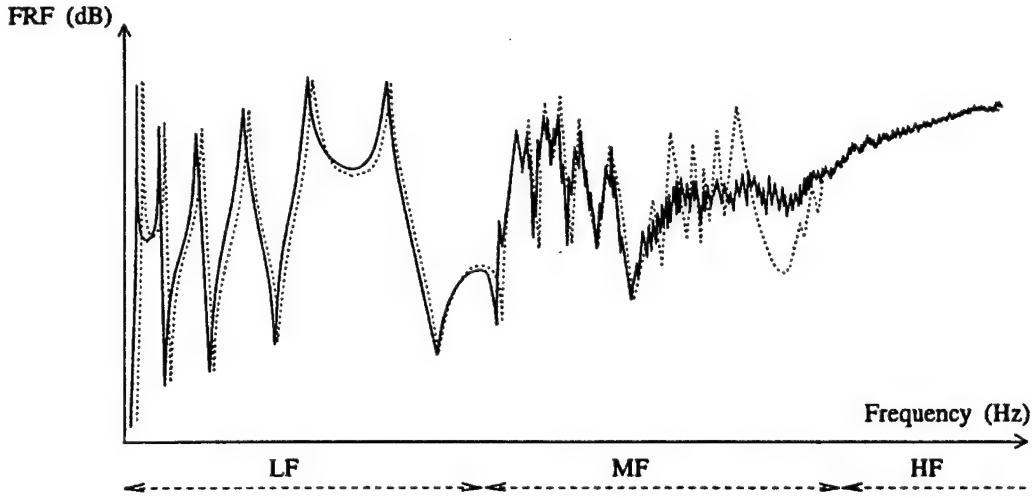


Fig. 8: From [8]: response of the master structure alone (solid line) and with fuzzy substructures attached (dotted line).

Note that Fig. 8 only shows the FRF of the master structure. The objective of the method is to predict the effect of fuzzy substructures on the response of the master structure—the response of the fuzzy substructures is not sought. The model thus employs a random boundary impedance operator, $\mathbf{Z}_{fuz}(\omega)$, which represents the effect of the fuzzy substructures on the master structure. This impedance is a sum of the impedances for the L fuzzy substructures:

$$\mathbf{Z}_{fuz}(\omega) = \sum_{l=1}^L \mathbf{Z}_{fuz}^l(\omega) \quad (6)$$

The equations of motion for the fuzzy structure are written as:

$$i\omega (\mathbf{Z}(\omega) + \mathbf{Z}_{fuz}(\omega)) \mathbf{U}(\omega) = \mathbf{f}(\omega) \quad (7)$$

where \mathbf{Z} , \mathbf{U} , and \mathbf{f} are the impedance, displacement, and forcing for the master structure. Note that the fuzzy substructures do not add degrees of freedom to the model. Soize developed a recursive method to find the random solution [8, 142, 143]. Alternatively, a solution can be found using Monte Carlo simulation [150].

4.1.2 Mid-Frequency Range Finite Element Method

In addition to fuzzy structure analysis, Soize et al. [142] introduced a numerical method suited to dynamic analysis of a structure in the mid-frequency range. The method was considered and extended by Vasudevan and Liu [152] and by Liu et al. [153] in 1991. (As an aside, the authors named Liu are two different researchers.) The method was also covered in the book by Ohayon and Soize [8], and a “gentle introduction” was offered by Sparrow [154]. Sparrow’s paper provides

a nice description of the approach and makes some insightful comments, so the interested reader may refer to it for a more complete overview.

Sparrow summarized Soize's method as a combination of traditional time integration and signal processing techniques [154]:

The mid-frequency range finite element method (MFR-FE) of Soize, as the method is denoted in the present paper, neither tries to do multiple large scale matrix solves or determine closely spaced eigenmodes in the MF region. The approach combines the traditional time integration methods available for low frequency time domain finite element simulation with standard demodulation/modulation signal processing (SP) techniques. It is the combination of the FE methods of numerical simulation with SP that forms the MFR-FE technique.[154]

Here, the method is also referred to as MFR-FE.

The basic approach of MFR-FE is to break up the mid-frequency region into a set of narrow frequency bands. For each narrow band, the mid-frequency response problem is transformed to the low-frequency range using a Fourier transform and demodulation. This leads to a low-frequency time-domain problem, which is solved using a conventional time integration technique. The solution is then sampled at a set of discrete points in time, and a Fourier transform and modulation are used to transform back to the solution for the mid-frequency band.

The derivation is now briefly summarized. For simplicity, a finite element discretization is assumed, but this is not restrictive. For a more general and detailed treatment, see chapter 7 of Ohayon and Soize [8].

To start, the mid-frequency range is considered as a union of narrow frequency bands. A narrow mid-frequency band is defined as:

$$\mathbb{B}_\nu = [\Omega_\nu - \Delta\omega/2, \Omega_\nu + \Delta\omega/2] \quad (8)$$

where Ω_ν is the center frequency and $\Delta\omega$ is the width of band \mathbb{B}_ν . In order for this band to be considered narrow, the condition is:

$$\Delta\omega/\Omega_\nu \ll 1 \quad (9)$$

Along with \mathbb{B}_ν , an associated low-frequency band is also defined:

$$\mathbb{B}_0 = [-\Delta\omega/2, \Delta\omega/2] \quad (10)$$

In addition, two time scales are introduced:

$$\tau_{\text{long}} = 2\pi/\Delta\omega \quad , \quad \tau_{\text{short}} = 2\pi/\Omega_\nu \quad (11)$$

Note that the low-frequency band, \mathbb{B}_0 , is associated with only one time scale, τ_{long} .

Next, the forcing vector, $\mathbf{F}(\omega)$, is taken to be of a special form:

$$\mathbf{F}(\omega) = \theta_\nu(\omega)\mathbf{B} \quad (12)$$

subject to the condition $\theta_\nu(\omega) = 0$ for frequencies outside of the band \mathbb{B}_ν . The inverse Fourier transform of $\theta_\nu(\omega)$ is a mid-frequency narrow band signal, $\theta_\nu(t)$. An associated low-frequency signal is defined by

$$\theta_0(t) = \theta_\nu(t)e^{-i\Omega_\nu t} \quad (13)$$

The Fourier transform of Eq. (13) is

$$\theta_0(\omega) = \theta_\nu(\omega + \Omega_\nu) \quad (14)$$

subject to the condition $\theta_0(\omega) = 0$ for frequencies outside of the band \mathbb{B}_0 .

Using the special form of the excitation given in Eq. (12), the equations of motion are

$$(-\omega^2[M] + i\omega[D(\omega)] + [K(\omega)])\mathbf{U}(\omega) = \theta_\nu(\omega)\mathbf{B} \quad (15)$$

Next, the damping and stiffness matrices are approximated as constant over the band \mathbb{B}_ν by taking their values at the center frequency

$$[D_\nu] = [D(\Omega_\nu)] \quad , \quad [K_\nu] = [K(\Omega_\nu)] \quad (16)$$

The equations of motion for the mid-frequency band are now written as

$$(-\omega^2[M] + i\omega[D_\nu] + [K_\nu])\mathbf{U}_\nu(\omega) = \theta_\nu(\omega)\mathbf{B} \quad (17)$$

where $\mathbf{U}_\nu(\omega)$ approximates the solution for the displacement field $\mathbf{U}(\omega)$. This approximation will be more accurate for a narrower frequency band.

Equation (17) can be written as

$$[A_\nu(\omega)]\mathbf{U}_\nu(\omega) = \theta_\nu(\omega)\mathbf{B} \quad (18)$$

where $[A_\nu(\omega)]$ is the dynamic stiffness matrix. Then the transform shown in Eq. (14) is used and also applied to the displacement, yielding the associated low-frequency equations of motion:

$$[A_\nu(\omega + \Omega_\nu)]\mathbf{U}_\nu(\omega) = \theta_0(\omega)\mathbf{B} \quad (19)$$

Taking the inverse Fourier transform yields the differential equations

$$[M]\ddot{\mathbf{U}}_0(t) + [\tilde{D}_\nu]\dot{\mathbf{U}}_0(t) + [\tilde{K}_\nu]\mathbf{U}_0(t) = \theta_0(t)\mathbf{B} \quad (20)$$

where

$$[\tilde{D}_\nu] = [D_\nu] + 2i\Omega_\nu[M] \quad , \quad [\tilde{K}_\nu] = -\Omega_\nu^2[M] + i\Omega_\nu[D_\nu] + [K_\nu] \quad (21)$$

Equation (20) can now be solved using a numerical time integration method. The time integration is performed from an initial time $t_i = m_i\Delta t$ to a final time $t_f = m_f\Delta t$, where m_i is a negative integer, m_f is a positive integer, and Δt is defined as

$$\Delta t = 2\pi/\Delta\omega \quad (22)$$

In addition, the initial conditions for the displacement and velocity are taken to be zero, and the time step for the integration is $\delta t = \Delta t/p$ where p is a positive integer.

After numerical integration, the low-frequency solution is time-sampled, the Fourier transform is taken, and the solution is transformed back to the mid-frequency band \mathbb{B}_ν . This yields the solution:

$$\mathbf{U}_\nu(\omega) \approx \Delta t \sum_{m=m_i}^{m_f} \mathbf{U}_0(m\Delta t) e^{-im\Delta t(\omega - \Omega_\nu)} \quad \forall \omega \in \mathbb{B}_\nu \quad (23)$$

The solution is only valid for this narrow band, such that

$$U_\nu(\omega) = 0 \quad \forall \omega \notin \mathbb{B}_\nu \quad (24)$$

The process can be repeated for many narrow bands to find an approximate solution for the mid-frequency range. This allows the response to be found by sampling narrow bands of frequency rather than sampling individual frequencies, which greatly improves the efficiency of a mid-frequency finite element analysis.

Note that the MFR-FE approach could be used for the master structure in a fuzzy structure, and in this sense it is a complement to the fuzzy structure analysis covered in section 4.1.1. However, it is separate from fuzzy structure theory, so it can be applied in a general sense to accelerate a mid-frequency vibration analysis. An example of this is the recent work of Savin [155], who applied the MFR-FE technique to the complex structure shown in Fig. 9 and compared the numerical results

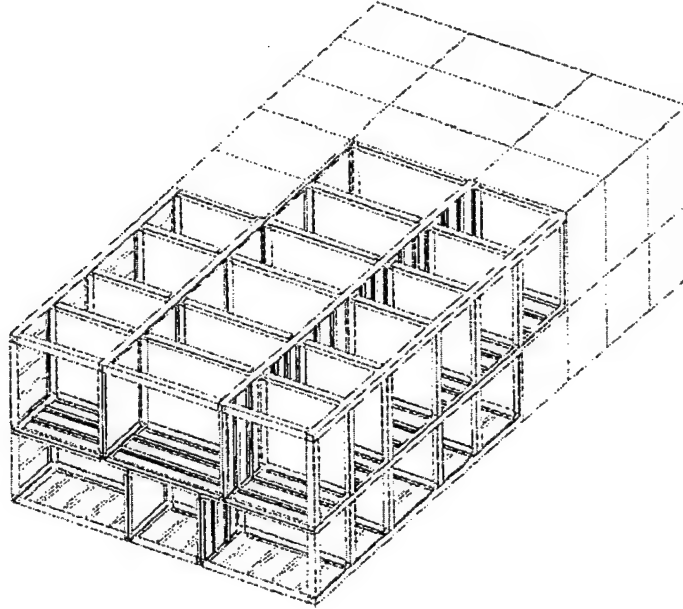


Fig. 9: From [155]: CAD drawing of the experimental structure, with several outer plates removed to show the internal geometry.

with experimental results. The finite element model had about 87,000 DOF, and the frequency range 100-1000 Hz was split into 24 narrow bands for the MFR-FE analysis. The FRF (acceleration and phase) at the excitation point for a harmonic point load is shown in Fig. 10. It can be seen that the numerical results match well with the experimental results up to 700 Hz. Furthermore, Savin noted that the MFR-FE analysis was about 10 times faster than conventional FEA. (Savin did not compare the accuracy of the numerical results with conventional FEA.)

Recently, Soize proposed a method for generating reduced models in the mid-frequency range [156, 157]. Arguing that a modal basis is not appropriate for the mid-frequency range, Soize introduced an energy operator associated with a frequency band. He showed that the dominant

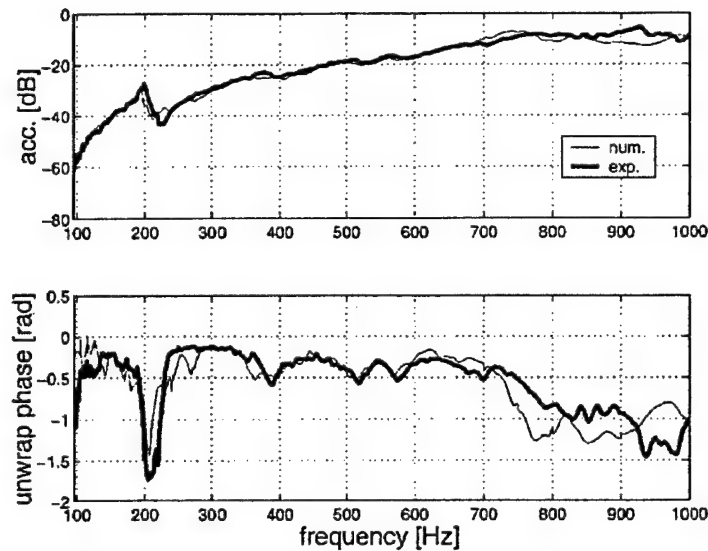


Fig. 10: From [155], comparison of numerical and experimental results for the response of the experimental structure at the excitation point for a harmonic point load.

eigenspace is spanned by the eigenfunctions associated with the highest eigenvalues of the energy operator. This allows a Ritz-Galerkin approach to be used to generate a reduced model for the frequency band. In 2000, Soize and Mziou [158] extended this technique by developing an alternative implementation of the Craig-Bampton component mode synthesis method [21] that employs the eigenfunctions of the energy operator rather than structural modes.

Soize seems to have been the first to recognize the need for mid-frequency analysis methods and the first to offer solutions for the mid-frequency gap. His work is notable in that the techniques were honed for specific problems, but they are still generally applicable to real engineering structures. The fuzzy structure theory seems well suited to handle certain classes of problems, although it will not fit the needs of many analysts. However, the mid-frequency range finite element method is compelling in that it is designed to accelerate FEA for mid-frequency analysis. Sparrow noted that the MFR-FE technique has been largely overlooked, and argued that it should be more widely adopted [154]:

...the ease with which the MFR-FE technique can be programmed implies that it should be able to be implemented in large scale commercial FEM programs with no problem. The MFR-FE method as applied to interior noise problems should be viewed as a supplement to low frequency modal methods and high frequency statistical energy methods, filling the gap in between. Soize's approach will allow us to make accurate structural vibration and interior noise predictions at much high[er] frequencies than we can now. [154]

As mid-frequency problems become more prevalent in automotive vehicles and other structures, Soize's contributions deserve to be re-visited.

4.2 Power Flow Analysis

In 1987, Cuschieri [17] specifically proposed using power flow analysis as a complement to FEA and SEA for the mid-frequency range:

In the mid-frequency range, a number of resonances exist which make FEA too costly and possibly not computationally feasible while SEA will only predict an average level of the response from which significant deviations can occur at the resonances of the structure. In this mid-frequency range an alternative is to use power flow techniques where the input and flow of vibrational power to excited and coupled structural components can be expressed in terms of input and transfer mobilities. [17]

He cited Pinnington and White [159] for the basic power flow technique. In a later paper [160], he referred to the technique as Mobility Power Flow (MPF), since mobility functions are used to capture the input power and the power flow between substructures. In the 1987 paper [17], the technique was demonstrated using an example of an L-shaped beam. Cuschieri showed that the power flow results tend to match the FEA results at low frequencies and are asymptotically equal to the SEA results at high frequencies. He argued that the approach has an advantage over FEA because substructuring is used to reduce the computational costs, and an advantage over SEA because the resonant structural response can be retained in the mobility functions.

In 1990, Cuschieri extended the MPF technique to periodic structures [161], using a multi-span beam as the example system, as well as to structures with line joints [160] rather than just point coupling, using an L-shaped plate as the example system. He also noted the advantages of the MPF approach:

The agreement with FEA results is good at low frequencies. However, the power-flow technique has an improved computational efficiency as the frequency increases. Compared to the SEA results, the power-flow results show a closer representation of the actual modal response of the coupled structure. [160]

McCollum and Cuschieri [162] also applied MPF to an L-shaped plate system with thick plates, with rotary inertia and shear deformation effects included in the analysis; and Cuschieri and McCollum [163] considered the power flow contributions of in-plane waves, which were neglected in previous studies of the L-shaped plate.

This sort of power flow approach may be a good strategy for the mid-frequency range as a compromise between FEA and SEA. However, the MPF technique is limited in that it is not formulated for general complex structures. An alternative is to use a finite-element-based reduced model for calculating the power flow, such as the methods covered in section 3.1.3. It should be emphasized that these power flow methods can be combined with uncertainty propagation analysis, in order to account for the effects of parameter uncertainties on the mid-frequency system response [39, 41, 74].

4.3 Variational Theory of Complex Rays

In 1996, Ladevèze [164] introduced the variational theory of complex rays (VTCR) for the mid-frequency vibration of weakly damped elastic structures. This paper was written in French, though an abridged English version was included. In a recent paper written in English [165], Ladevèze et

al. covered VTCR in more detail and provided some numerical examples. They characterize the method by describing three basic features, which are summarized as follows:

1. A new variational formulation of the reference problem is used, in which substructure approximations are independent and thus do not necessarily satisfy the displacement and stress transmission conditions at the interfaces between substructures. In [165], the reference problem is an assembly of two connected structures subject to harmonic vibration at a fixed frequency, ω .
2. "Two-scale approximations" are used, with a "slow" (long-wavelength) variable, \underline{X} , and a "fast" (small-wavelength) variable, \underline{Y} . The solution is assumed to be well described locally as the superposition of an infinite number of local vibration modes that satisfy the laws of dynamics for an infinite medium. The solution for the displacement is thus of the form $\underline{U}(\underline{X}, \underline{Y}, \underline{P}) = \underline{W}(\underline{X}, \underline{Y}, \underline{P}) \cdot \exp(i\omega \underline{P} \cdot \underline{Y})$ where \underline{P} is a vector characterizing the local vibration mode. The modes are defined explicitly in terms of the fast variable so that the unknowns are discretized amplitudes with relatively large wavelength.
3. From the calculated discretized amplitudes, only "effective quantities related to the elastic energy, kinetic energy, the dissipation work, etc." are retained.

In addition, Ladevèze et al. define "complex rays" to seek a solution for the mid-frequency vibration problem. They define interior, corner, and edge complex rays. The interior complex rays are "vibration modes in an infinite domain with the same mechanical properties" as the homogeneous substructure with which they are associated. A complex ray for the displacement may be expressed as:

$$\underline{U}(\underline{X}, \underline{Y}, \underline{P}) = \underline{W}(\underline{X}, \underline{P}) \cdot \exp(\omega \frac{\delta}{2} \underline{P} \cdot \underline{X}) \cdot \exp(i\omega \underline{P} \cdot \underline{Y}) \quad (25)$$

where the generalized amplitude \underline{W} is an n th-order polynomial in \underline{X} and δ is a small damping factor.

The authors refer to VTCR as a "true 'medium-frequency' method" because effective quantities are used for the time and space scales considered, rather than retaining "phenomena associated with small-length variations" that lead to "results which are very sensitive to data errors" [165]. In fact, the authors claim that the "spatial distribution of the solution has no 'physical meaning' from the mechanical point of view." Therefore, for quantity $q(\underline{X}, \underline{Y})$, a corresponding effective quantity is defined:

$$\tilde{q}(\underline{X}) = \frac{1}{4L^2} \int_{-L-X_1}^{L-X_1} dY_1 \int_{-L-X_2}^{L-X_2} dY_2 q(\underline{X}, \underline{Y}) \quad (26)$$

where L is the characteristic dimension of the domain of the substructure. This approach, which is the third feature listed above, seems to account for the averaging effects of uncertainties on the response in the mid-frequency range.

Three numerical examples are considered in [165]: a square plate, a triangular plate, and an assembly of 18 rectangular plates. For the square plate, the results at 600 Hz from modal analysis (1600 modes) and VTCR (64 DOF) are shown in Fig 11. The average displacement for the plate in the frequency range around 6000 Hz is plotted for each solution in Fig. 12. For the 18-plate assembly, the results are shown in Fig. 13 for the VTCR solution of the displacement and the effective displacement.



Fig. 11: From [165]: forced response of a simply-supported plate at 600 Hz calculated by modal analysis using 1600 modes (left) and VTCR using 64 DOF (right).

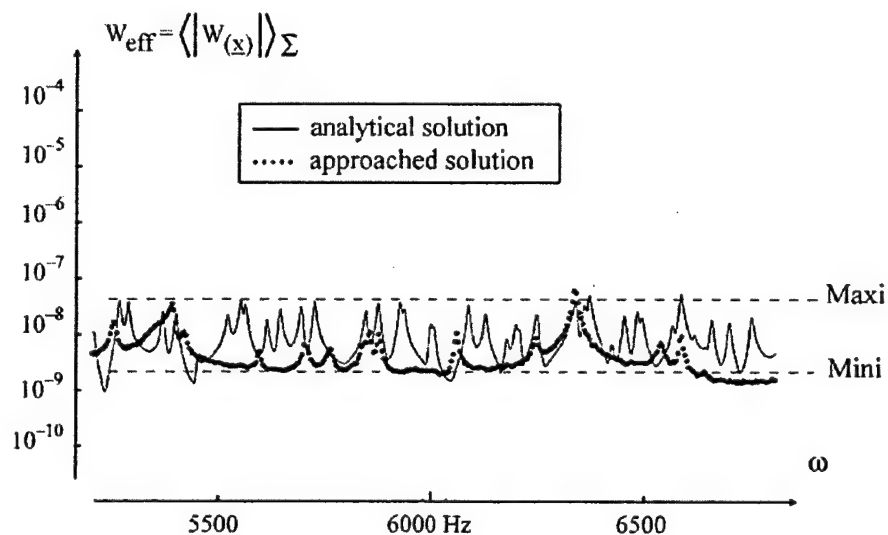


Fig. 12: From [165]: Results for the average plate displacement around 600 Hz calculated by modal analysis ("analytical solution") and VTCR ("approached solution").

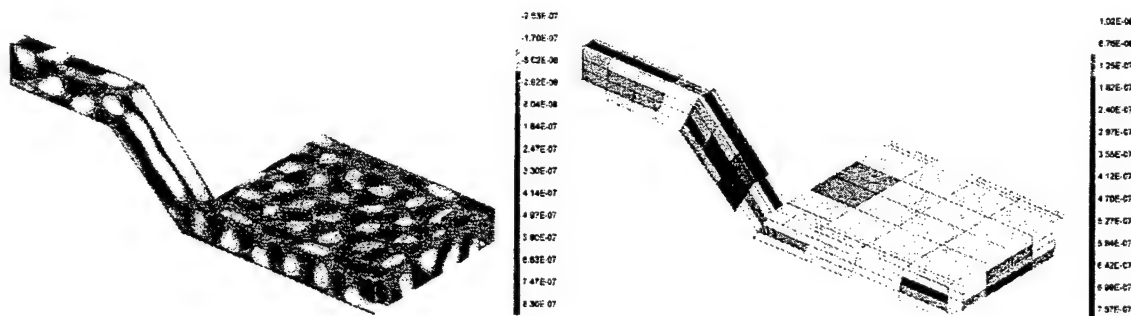


Fig. 13: From [165]: VTCR solution (left) and the post-processed effective quantities (right) for the vibration of a plate assembly.

The VTCR approach seems limited to systems with simple, homogeneous substructures. It appears that it would be difficult to apply the approach to a structure with a complex geometry, though it is possible that a library of basic substructures could be used, as is done in SEA codes. It should also be noted that the boundary conditions for the substructures are satisfied only in an average sense, which might have a detrimental impact on power flow approximations as well as on general modeling accuracy. However, more work needs to be done before the status or even the promise of this approach can be fully assessed.

4.4 Principal Components Analysis

Hasselmann and co-workers [18, 166] have introduced a probabilistic approach to vibration analysis, called principal components analysis, which combines elements of both FEA and SEA. In particular, in [18], the authors started with the equations of motion for two coupled multi-DOF subsystems modeled by the finite element method. Assuming stationary ergodic random excitation, "energy-type metrics" were derived from a "singular value decomposition of a frequency response matrix constructed by forming an array of discretized frequency response functions". The singular value decomposition leads to the form

$$H(\Omega) = \Phi D \gamma(\Omega) \quad (27)$$

where Φ is a matrix of "spatial mode shapes," D is a diagonal matrix of singular values, and γ is a matrix of "frequency response characteristics." It was shown that

$$HH^T = \Phi D^2 \Phi^T \quad (28)$$

The authors then stated:

...the trace of (HH^T) is related to the total energy, E_1 , of Component 1, and equals the sum of the elements of the diagonal matrix, D^2 . The elements of D are ordered from largest to smallest so that most of the energy is associated with the first few modes. Thus, the modal decomposition represented by [Eq. (27)] results in energy-based modes. The contention of this paper is that the energy-based modes constitute a more appropriate basis for characterizing structural response in the mid-to-high frequency range than the classical normal modes which are highly sensitive to modeling error. [18]

They quantified modeling uncertainty in terms of the "statistical differences between test measurements and analytical predictions based on a mathematical model," where the mathematical model was the finite element model. They then outlined a procedure for including the uncertainty in the frequency response analysis, employing metrics based on the correlation of modal analysis and test data. This approach was covered in detail in a more recent work by Hasselmann [167].

In [18], the authors applied principal components analysis to an example system of an L-shaped plate. They showed FRFs at nine locations for single-point excitation, as well as the corresponding first right eigenvector $\gamma_1(\Omega)$, for different frequency bands. One result is shown in Fig. 14. It can be seen that the eigenvector is representative of the FRFs for the system. In fact, the right eigenvectors are themselves FRFs; they are the principal FRFs ordered by decreasing energy contributions. For

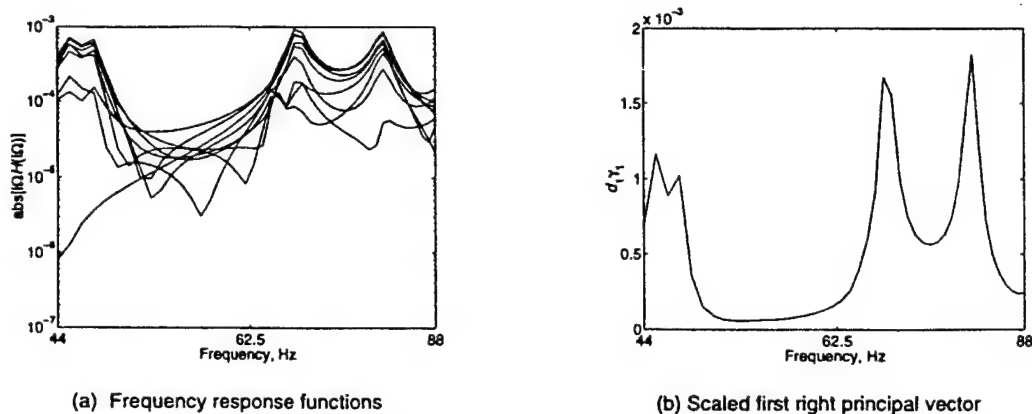


Fig. 14: From [18]: FRFs and the corresponding first principal eigenvector for an L-shaped plate.

this case, it was found that the corresponding energy represented by d_1^2 , the square of the first singular value, was over 80% of the total energy.

The authors concluded the development of the method by saying [18]:

The Principal Components approach ... places FEA and SEA on a common theoretical basis. Although different from classical SEA, the proposed approach is both statistical and energy-based, and therefore is considered to be a form of statistical energy analysis. [18]

More recently, Hasselman and co-workers [9] took the association of principal components analysis with SEA even further, when they introduced an approach called principal components-based statistical energy analysis (PC/SEA). In PC/SEA, the principal components method is used to determine damping and coupling loss factors for SEA. The purpose of this strategy is to take advantage of finely-meshed finite element models to improve the SEA results in the mid-frequency range. Two example models were considered, each with two connected plates: a flat plate configuration and an L-plate configuration. It was found that the PC/SEA results agreed well with benchmark numerical results, but differed by as much as an order of magnitude from the conventional SEA results. An important conclusion from this work was:

These results indicate superiority of finely-meshed finite element analysis combined with power flow methods for vibro-acoustic analysis in the mid-frequency range where neither conventional FEA nor conventional SEA have proven to be reliable in the past. [9]

It is interesting to note that although they adopted an SEA approach, Hasselman et al. emphasized the use of FEA as a starting point for an effective mid-frequency power flow analysis.

4.5 Hybrid FEA/SEA Formulations

In order to bridge the gap between FEA and SEA, it seems reasonable to seek a hybrid method in which some components are modeled by FEA and some by SEA. This type of approach was

pursued by Lu in a 1990 paper [168]. Considering a system of two components, he found that the main difficulty with using an FEA model for one substructure and an SEA model for the other was in trying to enforce displacement compatibility at the interface. In particular, he noted that phase information is not preserved in the SEA model, leading to more unknowns than equations in the combined formulation. Therefore, Lu imposed the power balance condition for the coupled system that the input power equals the dissipated power: $\Pi_{in} = \Pi_{damped}$. Then, he used an optimization technique with the design variables being the complex boundary forces and the objective function set to $\min(\Pi_{in} - \Pi_{damped})$. In this way, the displacements at the interface were forced to converge.

In 1999, Langley and Bremner [19] introduced an approach to modeling mid-frequency vibration that combines both a low-frequency deterministic approach for the global structure and a high-frequency SEA approach for certain substructures. In addition, elements of fuzzy structure theory [143, 146] and Belyaev's smooth function approach [106, 169, 170] are incorporated. Further developments of this hybrid technique have been reported by Shorter and Langley [171] and Shorter et al. [172]. The technique is called the Resound method [172] after the research consortium of the same name that led to its development.

Langley and Bremner took the view that the crux of the mid-frequency problem is the mixture of short- and long-wavelength response for structures that have beam-like frames with plate-like panels attached, such as the simple example shown in Fig. 15 [173]. In the mid-frequency range, the frame has long-wavelength behavior characterized by global vibration that can be modeled deterministically, whereas the attached plate has short-wavelength behavior characterized by local vibration that is significantly influenced by the effects of uncertainties. For illustration of this type of mixed vibration field, the 20th mode of the example system is shown on the right in Fig. 15.

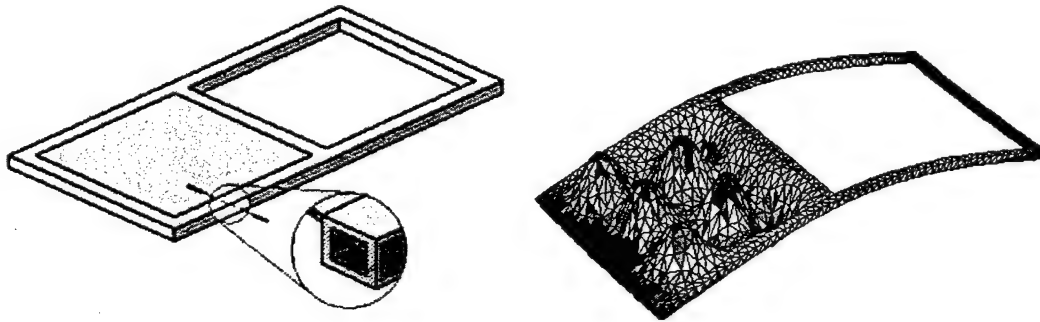


Fig. 15: From [173]: the frame-plate example system (left) and its 20th mode shape (right).

Langley and Bremner introduced a partitioning scheme based on wavelength to separate the degrees of freedom into a global set and a local set [19]:

The system response is partitioned into two components, corresponding to long wavelength and short wavelength deformations. The long wavelength component is modeled deterministically, while the short wavelength component is modeled by using SEA. The interaction between the two components is considered in some detail, and it is shown that results similar to those yielded by fuzzy structure theory are obtained. Two analogies with the previously mentioned work are drawn: the long wavelength

deformation can be identified with the master structure in fuzzy structure theory, or alternatively with the smooth response in the Belyaev approach. [19]

The use of deterministic modeling for the global structure with SEA modeling for the local substructures, with approximations of the interaction between the two types of components, comprises the framework of the Resound approach.

The basic derivation presented by Shorter et al. [172] is summarized as follows. Based on the wavelength partitioning scheme to separate global (g) and local (l) degrees of freedom, the equations of motion for a complex structure can be written as:

$$\begin{bmatrix} \mathbf{D}_{gg} & \mathbf{D}_{gl} \\ \mathbf{D}_{lg} & \mathbf{D}_{ll} \end{bmatrix} \begin{bmatrix} \mathbf{q}_g \\ \mathbf{q}_l \end{bmatrix} = \begin{bmatrix} \mathbf{f}_g \\ \mathbf{f}_l \end{bmatrix} \quad (29)$$

where \mathbf{D} is the dynamic stiffness matrix, \mathbf{q} is a vector of displacements, \mathbf{f} is a force vector.

In order to perform a deterministic analysis of the global system, the above equations are approximated by uncoupled sets of global and local equations of motion with appropriate perturbations due to the influence of the complementary subsystem:

$$[\mathbf{D}_{gg} - \Delta\mathbf{D}_{gg}] \mathbf{q}_g = \mathbf{f}_g - \Delta\mathbf{f}_g \quad (30)$$

$$\mathbf{D}_{ll} \mathbf{q}_l = \mathbf{f}_l - \Delta\mathbf{f}_l \quad (31)$$

where $\Delta\mathbf{D}_{gg}$ and $\Delta\mathbf{f}_g$ are the perturbations to the global structure caused by the local subsystem, and $\Delta\mathbf{f}_l$ is the perturbation to the forcing on the local subsystem due to the global subsystem.

The perturbation to element mn of the global dynamic stiffness matrix is:

$$(\Delta\mathbf{D}_{gg})_{mn} = \sum_j \alpha_j \beta_{j,mn} \quad (32)$$

where the summation is for all local component modes j in all short wavelength substructures. The term α is complex and accounts for frequency effects; it depends on the distribution of local natural frequencies, ω_j , about the excitation frequency, ω . The term β is real and accounts for spatial effects; it depends on the wavelength (or wavenumber) content of the local and global mode shapes.

The summation in Eq. (32) can be approximated by contributions from mass-controlled modes ($\omega_j < \omega$) and resonant modes ($\omega_j \approx \omega$). The contribution from stiffness-controlled modes is assumed to be small and is neglected. By assuming a uniform distribution of natural frequencies over the frequency range of interest, $\Delta\omega$, an average value of α can be obtained for the resonant and mass-controlled modes:

$$\langle \alpha \rangle_{res} \approx -i\pi\omega^3/2\Delta\omega \quad \langle \alpha \rangle_{mass} \approx -\omega^2 \quad (33)$$

It is interesting to note that since $\langle \alpha \rangle_{res}$ is imaginary, the resonant local modes provide damping to the global structure, which is in accordance with fuzzy structure theory [143]. (Lyon also showed that this fuzzy structure result can be retrieved with SEA [174].)

There are two approaches to the asymptotic evaluation of the term β . The first is a spatial correlation approach [171] that accounts for the expected spatial correlation between two points

in a mode when averaged over many local modes. The spatial correlation can be computed deterministically, though this can be computationally expensive. Asymptotic estimates may be derived by using a wave approach that neglects edge effects. In Fig. 16, the asymptotic and finite element results are shown for the correlation pattern of the resonant modes for the transverse displacement of a simply supported rectangular plate. The edge effects can be seen in the finite element results. The second approach to the asymptotic evaluation of β is to use an asymptotic modes approach, which has given similar results to the spatial correlation method for validation examples [172].

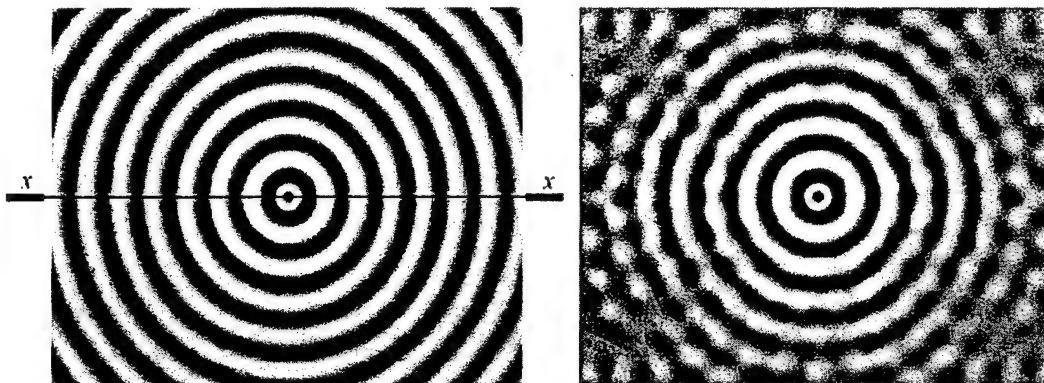


Fig. 16: From [171]: spatial correlation patterns of the resonant modes for transverse displacement of a simply supported rectangular plate, asymptotic results (left) and finite element results (right).

The above accounts for the perturbation to the global dynamic stiffness matrix due to the local modes of the fuzzy subsystems. The perturbation to the global forcing, Δf_g , accounts for additional forcing on the global structure from the generalized forces acting on the local coordinate set. Various approximations were covered in [19]. One method for obtaining the modified global forcing is to estimate the “blocked” local response, or the response of the local subsystem when the global response is held fixed.

The perturbation to the local forcing, Δf_l , is more straightforward. Recall that the local resonant modes lead to a damping effect in the global response, which is consistent with fuzzy structure theory. The energy that leaves the global degrees of freedom due to this damping effect is the energy that is injected into the local system from the global system.

Therefore, the local response can be solved using SEA, where the input power for the local system is the sum of the direct input power on the local coordinates plus the power transmitted from the global system due to fuzzy damping effects. The global response is obtained by inverting the modified dynamic stiffness matrix of Eq. (30).

To date, the Resound method has only been validated for simple example systems such as coupled rods [19] and plate structures [171]. It remains to be seen if this approach can be applied systematically to general complex structures.

4.6 Hybrid Conventional/Energy FEA

In the last section, hybrid FEA/SEA techniques were reviewed. If one is to employ such a hybrid approach in which some parts of the structure are modeled with FEA and some are modeled with a high-frequency method, another good candidate for the high-frequency method is Energy FEA [109–113, 123]. EFEA is arguably a more natural fit with conventional FEA, since it is a finite element formulation.

In 1999, Vlahopoulos and Zhao [125] presented a hybrid conventional/energy FEA method for mid-frequency structural vibration. In this hybrid formulation, it is assumed that a complex structure is divided into “long” components that have relatively high-frequency vibration, and “short” components that have relatively low-frequency vibration. The terms “long” and “short” refer to the scale of the characteristic length of the component relative to the wavelength of vibration: a short component has only a few wavelengths over its length, while a long component has many wavelengths.

In their initial development [125], Vlahopoulos and Zhao considered only systems of coupled, colinear beams with the excitation applied on long components, such as the system shown in Fig. 17. The key challenge was in capturing the energy transfer at junctions between long and

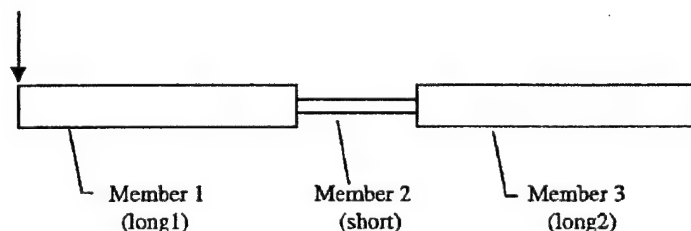


Fig. 17: From [125]: system of long and short beams with excitation on a long component.

short components. They handled this by relating the displacement and slope in the conventional finite element formulation to the amplitude of the impinging wave in the energy finite element formulation for each junction between long and short components. This so-called hybrid joint leads to the EFEA power transfer coefficients at long-short junctions that complement the power transfer coefficients at long-long junctions. The latter are calculated analytically by modeling a long component as a semi-infinite structure. The solution process was then to calculate the response of the long members first, and then calculate the response of the short members, subject to incoherent excitation at the short-long joints, using conventional FEA. A flow chart illustrating the hybrid FEA/EFEA computational process is shown in Fig. 18. For the systems considered, the hybrid results showed good agreement with analytical results. In contrast, the results from EFEA applied to all components were inaccurate, because the resonant behavior of the short components was not captured by EFEA alone.

In 2000, Zhao and Vlahopoulos [126] extended this approach by allowing for the case of excitation applied to short members. This required enhancing the computational process by including an iterative solution for the interface system of equations in the hybrid joint formulation. After convergence of the solution for the hybrid interfaces, the power flow into the long components and the boundary conditions for the short components were used to solve the system response.

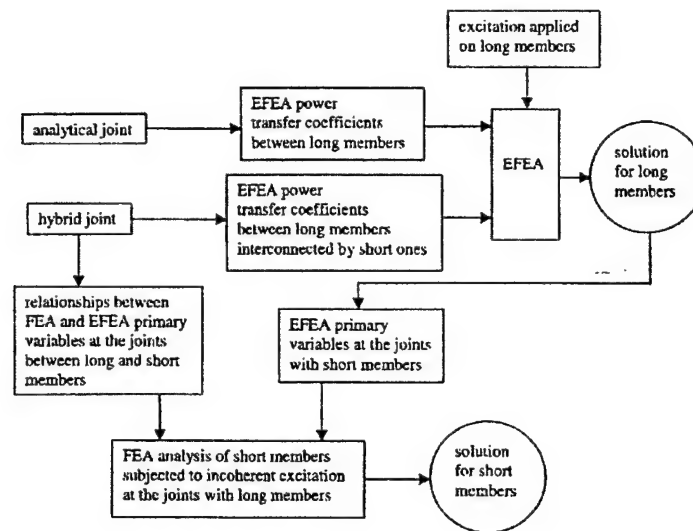


Fig. 18: From [125]: flow chart of the hybrid FEA/EFEA computational process for a system with excitation on long components.

In 2001, Vlahopoulos and Zhao [127] used the hybrid FEA formulation to investigate power flow in systems of colinear beams. They examined phenomena such as power re-injection [175, 176], when power reflected from a joint back into a short component eventually impinges the joint again and is partially transmitted. The hybrid method showed good agreement with analytical solutions.

To date, this hybrid FEA method has only been applied to systems of colinear beams. However, Zhao and Vlahopoulos noted that in the theoretical development “no assumptions are made that would prohibit the extension of this work to members with multiple types of waves or to members connected at arbitrary angles” [126]. Nevertheless, the process of mating conventional and energy FEA formulations at hybrid junctions could become cumbersome for more complex connections, and it remains to be seen whether this approach can be extended to arbitrary connections between components in general complex structures.

5 COMMENTS ON THE STATE OF THE ART

Considered as a whole, some clear trends can be seen in mid-frequency structural dynamics research to date. First, power flow is a recurring theme, and there is a growing consensus that some form of power flow analysis should be employed, at least as a complement to a more traditional vibration analysis. Second, most mid-frequency methods to date are substructure-based approaches. Of course, this is not surprising, since the nature of a complex structure is such that it is defined as an assembly of simpler structures. In addition, analysis techniques are inherently more accurate for predicting the vibration response of components than the response of combinations of components.

In fact, it appears that the major modeling issues for the mid-frequency range are primarily related to the coupling between the components of a complex structure. Consider these examples from the array of methods that have been reviewed:

- Hybrid methods are focused on enforcing compatibility between different modeling methods at component interfaces, and the attendant complications have limited these techniques to simple structures
- The applicable range of component mode synthesis models can be pushed into the mid-frequency region via component interface reduction methods, which increase the computational efficiency and enable power flow analysis
- The fidelity of a wave-based approximation depends largely on the handling of scattering and transmission of waves at substructure junctions
- An important—and limiting—premise of the variational theory of complex rays is that the component boundary conditions are satisfied only in an approximate sense

Thus, it is seen that the treatment of component interfaces constitutes the major challenge for most mid-frequency techniques, and the accuracy and applicability of each method is highly correlated with how the interface issues are handled.

There is no clear leader among mid-frequency techniques presented to date, though several are promising. However, it should be noted that significant research progress has been made in the area of reducing the cost for finite-element-based analysis, thus increasing the range of application up into the mid-frequency range. Notable methods in this area include the mid-frequency range finite element method of Soize [8, 142, 154], the automated multi-level substructuring technique of Bennighof [65–71], and the characteristic constraint mode approach of Pierre [36–40, 75]. These methods address a second key challenge posed by the mid-frequency range, that of handling the computational difficulties associated with this region.

In addition to interface and computational issues, the effect of parameter uncertainties is a significant concern in the mid-frequency range. This is the third main area of difficulty for mid-frequency analysis. One observation on this front is that information should not be thrown out too early in the process by imposing broad assumptions from the start. For instance, phase information and/or spatial distribution of energy within each substructure may not be kept in the final results, but it may still be a crucial factor for determining the overall response before averaging out due to uncertainties.

Overall, most mid-frequency methods to date seem rather limited in their scope of application. Many are restricted to very simple models, such as beams and plates. The need for methods that

may be used for general complex structures was pointed out by Savin, who noted that "the proposed numerical methods should be applicable to arbitrary, complex configurations and topologies that exhibit the most characteristic features and difficulties raised by midfrequency vibration predictions" [155]. In assessing the state of the art, one must agree that there is a surprising lack of even basic mid-frequency analysis tools for complex structural systems. It is clear that more research effort must be placed on developing mid-frequency methods that can be employed by engineers for complex structures.

6 FUTURE RESEARCH DIRECTIONS

Based on the issues and challenges noted above, some ideas for future research are now presented. This list of topics and suggestions is by no means exhaustive. Nevertheless, it is hoped that this coverage will help spark further work and new initiatives.

6.1 Tailored Finite Element Models and Methods

To start, it is worth considering what can be done in finite element analysis in order to help enable the treatment of mid-frequency problems. After all, one reason that FEA is so successful is that one provides basic information about the system—geometry, material properties, forcing—and then there is a very systematic way to generate and solve the corresponding numerical model. Furthermore, there is an excellent foundation of commercial software for performing the analysis.

Certainly, some specialized finite element methods already exist. Notable examples include power flow / energy FEA of Nefske and Sung [109] and the mid-frequency range FEA of Soize [8, 142, 154]. Implementations and further developments of these techniques should be examined. In addition, other approaches to narrow-band, energy-based, or wave-based FEA should be pursued.

More fundamental issues should also be explored, such as the use of alternative elements with local basis functions tailored to vibration modeling, and the incorporation of probabilistic analysis related to the effects of parameter uncertainties. Another option is to increase the superelement capabilities of existing software. Just as traditional FEA is based on elements and assembly of global equations from local representations, mid-frequency FEA could be based on superelements and coupling of superelement models at interfaces. Alternatively, decomposition and reduction techniques could be used to essentially post-process finite element models for the mid-frequency range. This is discussed next.

6.2 Domain Decomposition and Reduced Order Modeling

It is natural to treat a complex structure as an assembly of substructures. However, partitioning a finite element model or some other geometric representation into component structures is not necessarily straightforward or easy. There are already many domain decomposition techniques, but these are likely to favor partitioning into components based on factors such as minimal connectivity or similar component sizes. Instead, it would be preferable to develop a physics-based domain decomposition technique for mid-frequency analysis. Such an approach could be designed to have some important advantages. First, it would identify each component based on an assessment of the subdomain characteristics, thus enabling an appropriate modeling technique for that substructure as well as enhancing the convergence of the solution. Second, it would help ensure that the results are correlated with the parameters of key components of the system. Third, it could be tailored to a particular hybrid or specialized mid-frequency analysis approach for the full structure.

In addition to decomposition, more work can be done in reduced order modeling. Finite-element-based reduced order modeling and power flow analysis methods are very promising. Of course, as computers become faster and more powerful, the frequency limits of FEA will be pushed higher, and the need for reduced models may be questioned. However, specialized and approximate methods that yield reduced models will always have a place in mid-frequency analysis, and in structural dynamics analysis in general. These methods increase the simulation space even for the

fastest computers, thereby increasing the design space. Furthermore, these methods are important because they enhance understanding of the physics of the system. They allow one to isolate key system parameters, to distill and interpret the results, and to consider parameter uncertainties and the propagation of uncertainty in the response. These are all crucial to evaluating and improving a design. Thus, reduced order modeling methods enable virtual prototyping and design optimization, including for the mid-frequency range.

Finally, a combination of decomposition and reduced modeling techniques can be envisioned. In particular, for mid-frequency methods that are applicable only to systems with simple substructure models, the decomposition could include identification of the subsystem model type and the corresponding model parameters for each substructure. This would be useful as a pre-processor for any system modeling software that employs a library of subsystems, such as a commercial SEA code.

6.3 Smart Hybrid Methods

In a similar vein to reduced models, hybrid models are a natural fit for the mid-frequency range. In fact, several hybrid mid-frequency methods have been reviewed. One interesting possibility would be to develop "smart" hybrid methods that could be combined with a domain decomposition technique to automatically assign one of two or more modeling approaches for each identified component. This assignment could be varied according to the frequency range of interest, loading conditions, and so forth. Savin [155] noted that the dynamic behavior changes in the mid-frequency range, and therefore varying a combination of models may be required:

In the intermediate-frequency range there is a transition between two dynamic behavior patterns; therefore, there should be a transition of numerical models as well. [155]

It should be noted that the Resound method [19, 172] does incorporate a system for identifying the "fuzzy" substructures according to frequency. As the frequency of vibration is increased, more substructures are treated as fuzzy substructures [172]. One can envision a more comprehensive implementation of this approach, though, incorporating the decomposition methods suggested above as well as special interface models.

6.4 Improved Modeling of Interfaces and Joints

As mentioned earlier, the primary modeling challenges of the mid-frequency range appear to be those related to the junctions between component structures. Therefore, improved modeling of interfaces between substructures must be considered a crucial area for future research. Examples include:

- the treatment of transmission and scattering of waves at component interfaces
- the hybridization of two different approaches for substructure modeling
- the detailed representation of joints or complex connections that may include friction and other nonlinear effects

Certainly, the first two examples are related to the limiting factors of several mid-frequency methods presented to date. In particular, for hybrid methods, the different modeling methods must be resolved through an appropriate interface model. This seems to be a rich topic for further work, with potential for big payoffs in engineering applications.

With respect to the third item, it is clear that complex connections and joints between substructures have not received much attention in the mid-frequency literature. This is probably due to the fact that, even with ideal representations of interfaces, the inherent discontinuities at junctions lead to significant complexities in the analysis. However, it is important to realize that in the mid-frequency range, the wavelengths can be sufficiently small that additional modeling challenges are encountered at interfaces. For example, in recent experimental studies, Buehrle et al. [7] and Savin [155] noted modeling discrepancies at higher frequencies due to local effects around rivets. Buehrle et al. pointed out the need for refined interface modeling:

Predicted frequencies for the plate stiffener model with rivet line attachment were consistently lower than the measured natural frequencies. This indicates that further refinement of the model of the attachment interface is required. [7]

In addition, Cudney [177] made this general comment about the treatment of joints:

Overall, a problem which plagues solution of mid- to high-frequency structural dynamics design and analysis problems is that our modeling is greatly influenced by our assumptions about damping at joints. The response predictions are highly sensitive to these modeling assumptions. [177]

Thus, it is clear that this topic should receive more attention in the future.

One strategy would be to conduct experimental studies for specific types of joints. Then, empirical results could be combined with substructure models as a sort of hybrid approach. A similar strategy would be to treat joints and interfaces as special classes of substructures. Specialized models could then be incorporated into "part libraries" for a given analysis technique. This type of approach is already employed for subsystem modeling in SEA software.

6.5 Applications and Benchmarks

As noted earlier, mid-frequency techniques have mostly been applied only to simple structures, such as assemblies of beams or plates. There is a need for more realistic applications in order to test the capabilities of existing techniques, to assess the promise of various approaches, and to identify critical unresolved issues as well as possible solutions. Numerical and/or experimental studies for realistic complex structures in the mid-frequency range should be one of the top priorities for future research, at least as part of a larger development effort. It should be noted that there has been some progress in this area in the last few years, such as the numerical/experimental studies of Buehrle et al. [7] and Savin [155], and the applications to vehicle structures by Pierre and co-workers [38, 40, 75].

In a similar vein, the need for mid-frequency benchmarks was pointed out by Cudney [177] in summarizing the discussions at the "Structural Dynamics @ 2000" forum:

What must be done? We should develop systematic benchmarks of predictions of modes and response, and communicate these to educators in the field. [177]

A set of benchmarks, perhaps ranging from simple structures to representative complex structures, would help establish the accuracy, applicability, and scalability of mid-frequency techniques.

6.6 Optimized Methods for Special Classes of Structures

In keeping with the theme of applications described above, it should be emphasized that certain classes of complex structures can be handled by methods tailored for the unique characteristics of the system. Thus, optimized methods for certain classes of structures should be pursued where possible. A notable example of this is the recent work on reduced order modeling of the vibration of mistuned bladed disks [178–182]. Bladed disks used in turbomachinery (e.g., jet engine rotors) can suffer significant vibration problems under certain operating conditions due to the small discrepancies among the blades, called mistuning. By many standards, this is a mid-frequency vibration problem: the vibration problems are usually in a region of high modal density, and may occur beyond the first several hundred system modes, yet individual system modes are still of interest; the small uncertainties in the system have a large effect on the response and must be considered for meaningful predictions; and the computational costs of using FEA to predict the statistics of the mistuned response are prohibitively high. However, by taking advantage of the cyclic symmetry of the nominal system, reduced models can be developed that provide efficient yet accurate predictions of the free and forced response. There may be other types of structures—or substructures, connected components, joints, etc.—that would benefit from a similarly customized implementation of mid-frequency vibration analysis.

6.7 A New Approach

Ultimately, an ideal mid-frequency technique could be imagined as a fundamentally new approach for the mid-frequency range that serves the same kind of role as FEA in the low-frequency range and SEA in the high-frequency range. While it is difficult to speculate on the details of such an ultimate mid-frequency (UM) technique, it is possible to consider some general themes that might be incorporated. For instance, the UM technique would take good ideas from both FEA and SEA. From FEA: clear requirements for representing the system, systematic approach to model generation, and capture of appropriate details/resonances in the solution. From SEA: calculation of power flow between subsystems, simple representation of results in terms of key system parameters, and accounting for uncertainty. Furthermore, the UM technique might embrace the inherent wave-mode duality in the mid-frequency range, positioning it as a hybrid or pseudo-hybrid method. Finally, the UM technique would most likely be able to handle low-frequency and high-frequency analysis, although it would lose accuracy relative to FEA at the low end and lose efficiency relative to SEA at the high end of the frequency spectrum.

7 CONCLUSIONS

In this paper, approaches for handling the mid-frequency vibration analysis of complex structures were reviewed. It was found that the mid-frequency range poses significant challenges in three main areas:

- **Connection:** The treatment of a complex structure as an assembly of connected component structures is both a natural starting block and a natural stumbling block in the mid-frequency range. For instance, different component structures may be best described by qualitatively different models for the same load case: one substructure may have relatively high modal density, the other low modal density. Also, the wave-mode duality of structural vibration is especially important in the mid-frequency range. Incompatibilities between a modal approach and a wave- or energy-based approach must be resolved if one is to consider hybrid methods that employ both types of analysis. Even if a hybrid approach is not taken, each interface between components involves vibration, wave, and energy transmission issues that are not easily resolved. Furthermore, complicated and/or jointed connections require specialized modeling. In terms of problem formulation, the partitioning of the model into appropriate substructures is a crucial step and may not always be obvious or convenient, particularly with respect to specialized mid-frequency analysis methods.
- **Computation:** There is a critical accuracy versus efficiency trade-off in the mid-frequency range. From a low-frequency perspective, refining a finite element mesh, running the finite element analysis, and extracting key results from the analysis can be prohibitively expensive as frequency increases. Therefore, approximate methods must be adopted. From a high-frequency perspective, the situation is reversed. The simplifying assumptions that enable efficient analysis methods in the high-frequency range may not be appropriate for the mid-frequency range, leading to a quantitative and even qualitative breakdown in modeling accuracy as frequency decreases.
- **Prediction:** As the frequency of vibration increases from the low- to mid-frequency range, parameter uncertainties have a greater influence on the response, especially as the wavelength decreases to the scale of random structural variations (e.g., manufacturing tolerances). At some point in the mid-frequency range, a deterministic model represents at best one member in the population of structures with the same nominal design, such that uncertainty in the system must be considered in order to predict the response. Estimated parameters and unmodeled structural complexities provide additional sources of uncertainty. Furthermore, from an engineering perspective, it is important not only to predict the response for a particular design, but also to predict the effect of design changes on that response.

Of these three main areas, it was determined that the primary challenges of the mid-frequency range are related to the first area, the handling of substructure connections. It was also found that the best progress to date has been achieved in the second area—specifically, in improving the computational efficiency of finite-element-based analysis so as to push its range of application higher, into the mid-frequency range.

A major shortcoming of most mid-frequency techniques is that they are either restricted to simple structures, or they cannot be readily applied to realistic complex structures. Therefore, it is

clear that more effort needs to be expended on delivering tools that can be employed by engineers to solve real problems. This does not imply that fundamental research in this area is not important or instructive; rather, basic research must be complemented by and/or guided by applications. In addition, it is important to consider how a method enables not only the analysis but also the design of complex structures.

Overall, when evaluating the potential of a mid-frequency technique, it is good to keep in mind the observation made by G. E. P. Box on scientific model building: "All models are wrong but some are useful" [183]. In the present context, "useful" is taken to mean that a model can be applied systematically to the analysis of a complex structure in order to predict the vibration response, to evaluate the design, and to assess the effect of design changes. Ultimately, the structural dynamics research community must meet a single engineering challenge: finding mid-frequency modeling and analysis techniques that are useful.

References

- [1] Ibrahim, R. A., 1987, "Structural Dynamics With Parameter Uncertainties," *Applied Mechanics Reviews*, **40**, pp. 309–328.
- [2] Hodges, C. H., and Woodhouse, J., 1986, "Theories of Noise and Vibration Transmission in Complex Structures," *Reports on Progress in Physics*, **49**, pp. 107–170.
- [3] Ewins, D. J., and Inman, D. J., 2001, "Introduction to SD2000: Origins, Concepts, Plans," *Structural Dynamics @ 2000: Current Status and Future Directions*, Research Studies Press Ltd., Hertfordshire, England, pp. 1–7.
- [4] Inman, D. J., and Ewins, D. J., 2001, "The Outcome of SD2000," *Structural Dynamics @ 2000: Current Status and Future Directions*, Research Studies Press Ltd., Hertfordshire, England, pp. 445–448.
- [5] Nack, W. V., 2000, "Computational Noise and Vibration in the Automotive Industry," *Proceedings of the 41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, Atlanta, GA, **5**, pp. 15–23.
- [6] Kompella, M. S., and Bernhard, R. J., 1996, "Variation of Structural-Acoustic Characteristics of Automotive Vehicles," *Noise Control Engineering Journal*, **44**, pp. 93–99.
- [7] Buehrle, R. D., Fleming, G. A., Pappa, R. S., and Grosveld, F. W., 2001, "Finite Element Model Development for Aircraft Fuselage Structures," *Sound and Vibration*, **35**, pp. 32–38.
- [8] Ohayon, R., and Soize, C., 1998, *Structural Acoustics and Vibration: Mechanical Models, Variational Formulations and Discretization*, Academic Press, San Diego, CA.
- [9] Hasselman, T., Yap, K., Yan, H., and Parrett, A., 2002, "Statistical Energy Analysis by Principal Components for Mid-Frequency Analysis," *Proceedings of the 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, American Institute of Aeronautics and Astronautics.
- [10] Lyon, R. H., and Maidanik, G., 1962, "Power Flow Between Linearly Coupled Oscillators," *Journal of the Acoustical Society of America*, **34**, pp. 623–639.
- [11] Smith, Jr., P. W., 1962, "Response and Radiation of Structural Modes Excited by Sound," *Journal of the Acoustical Society of America*, **34**, pp. 640–647.
- [12] Scharton, T. D., and Lyon, R. H., 1968, "Power Flow and Energy Sharing in Random Vibration," *Journal of the Acoustical Society of America*, **43**, pp. 1332–1343.
- [13] Lyon, R. H., 1975, *Statistical Energy Analysis of Dynamical Systems: Theory and Applications*, MIT Press, Cambridge, MA.
- [14] Keane, A. J., and Price, W. G., eds., 1994, *Statistical Energy Analysis: An Overview, with Applications in Structural Dynamics*, Cambridge University Press, Cambridge, UK.

- [15] Lyon, R. H., and DeJong, R. G., 1995, *Theory and Application of Statistical Energy Analysis*, Butterworth-Heinemann, Boston, MA.
- [16] Lalor, N., 1996, "Statistical Energy Analysis and its Use as an NVH Analysis Tool," *Sound and Vibration*, **30**, pp. 16–20.
- [17] Cuschieri, J. M., 1987, "Power Flow as a Complement to Statistical Energy Analysis and Finite Element Analysis," *Statistical Energy Analysis*, American Society of Mechanical Engineers, New York, NY, NCA-3, pp. 55–61.
- [18] Hasselman, T. K., Anderson, M. C., and Lai, Y.-C., 1998, "Linking FEA and SEA by Principal Components Analysis," *Proceedings of the 16th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **2**, pp. 1285–1291.
- [19] Langley, R. S., and Bremner, P., 1999, "A Hybrid Method for the Vibration Analysis of Complex Structural-Acoustic Systems," *Journal of the Acoustical Society of America*, **105**, pp. 1657–1671.
- [20] Hurty, W. C., 1965, "Dynamic Analysis of Structural Systems Using Component Modes," *AIAA Journal*, **3**, pp. 678–685.
- [21] Craig, R. R., and Bampton, M. C. C., 1968, "Coupling of Substructures for Dynamic Analyses," *AIAA Journal*, **6**, pp. 1313–1319.
- [22] Goldman, R. L., 1969, "Vibration Analysis by Dynamic Partitioning," *AIAA Journal*, **7**, pp. 1152–1154.
- [23] MacNeal, R. H., 1971, "A Hybrid Method of Component Mode Synthesis," *Computers and Structures*, **1**, pp. 581–601.
- [24] Rubin, S., 1975, "Improved Component-Mode Representation for Structural Dynamic Analysis," *AIAA Journal*, **13**, pp. 995–1006.
- [25] Hintz, R. M., 1975, "Analytical Methods in Component Mode Synthesis," *AIAA Journal*, **13**, pp. 1007–1016.
- [26] Craig, R. R., and Chang, C.-J., 1976, "Free-Interface Methods of Substructure Coupling for Dynamic Analysis," *AIAA Journal*, **14**, pp. 1633–1635.
- [27] Hale, A. L., and Meirovitch, L., 1980, "A General Substructures Synthesis Method for Dynamic Simulation of Complex Structures," *Journal of Sound and Vibration*, **69**, pp. 309–326.
- [28] Craig, R. R., *Structural Dynamics: An Introduction to Computer Methods*, ch. 19. New York, NY: John Wiley & Sons, 1981.
- [29] Craig, R. R., 1995, "Substructure Methods in Vibration," *ASME Journal of Mechanical Design*, **117B**, pp. 207–213.

- [30] Seshu, P., 1997, "Substructuring and Component Mode Synthesis," *Shock and Vibration*, **4**, pp. 199–210.
- [31] Shyu, W.-H., Ma, Z.-D., and Hulbert, G. M., 1997, "New Component Mode Synthesis Method: Quasi-Static Mode Compensation," *Finite Elements in Analysis and Design*, **24**, pp. 271–281.
- [32] Shyu, W.-H., Gu, J., Hulbert, G. M., and Ma, Z.-D., 2000, "On the Use of Multiple Quasi-Static Mode Compensation Sets for Component Mode Synthesis of Complex Structures," *Finite Elements in Analysis and Design*, **35**, pp. 119–140.
- [33] Craig, R. R., and Chang, C.-J., "Substructure Coupling for Dynamic Analysis and Testing," NASA Report CR-2781, Washington, DC, 1977.
- [34] Guyan, R. J., 1965, "Reduction of Stiffness and Mass Matrices," *AIAA Journal*, **3**, p. 380.
- [35] Brahmi, K., Bouhaddi, N., and Fillod, R., 1995, "Reduction of Junction Degrees of Freedom in Certain Methods of Dynamic Substructure Synthesis," *Proceedings of the 13th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **2**, pp. 1469–1475.
- [36] Castanier, M. P., Tan, Y.-C., and Pierre, C., 2001, "Characteristic Constraint Modes for Component Mode Synthesis," *AIAA Journal*, **39**, pp. 1182–1187.
- [37] Tan, Y.-C., Castanier, M. P., and Pierre, C., 2000, "Characteristic-Mode-Based Component Mode Synthesis for Power Flow Analysis in Complex Structures," *Proceedings of the 41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, American Institute of Aeronautics and Astronautics, Reston, VA, **1**, pp. 908–917.
- [38] Tan, Y.-C., Castanier, M., and Pierre, C., 2001, "Efficient Reduced Order Modeling of Low- to Mid-Frequency Vibration and Power Flow in Complex Structures," *Proceedings of the 19th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **2**, pp. 1070–1076.
- [39] Tan, Y.-C., Castanier, M. P., and Pierre, C., 2001, "Statistical Approximations for Characteristic-Mode-Based Power Flow Analysis," *42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, American Institute of Aeronautics and Astronautics, Reston, VA, **4**, pp. 3039–3049.
- [40] Tan, Y.-C., Lee, S.-Y., Castanier, M. P., and Pierre, C., 2001, "Efficient Component-Based Vibration and Power Flow Analysis of a Vehicle Structure," *Proceedings of the ASME Noise Control and Acoustics Division — 2001*, American Society of Mechanical Engineers, New York, NY, NCA-28, pp. 103–112.
- [41] Tan, Y.-C., 2001, "Efficient Modeling of Low- to Mid-Frequency Vibration and Power Flow in Complex Structures," Ph.D. thesis, The University of Michigan, Ann Arbor, MI.

- [42] Bourquin, F., and d'Hennezel, F., 1992, "Intrinsic Component Mode Synthesis and Plate Vibrations," *Computers and Structures*, **44**, pp. 315–324.
- [43] Bourquin, F., and d'Hennezel, F., 1992, "Numerical Study of an Intrinsic Component Mode Synthesis Method," *Computer Methods in Applied Mechanics and Engineering*, **97**, pp. 49–76.
- [44] Bouhaddi, N., and Fillod, R., 1992, "Substructuring Using a Linearized Dynamic Condensation Method," *Computers and Structures*, **45**, pp. 679–683.
- [45] Bouhaddi, N., and Fillod, R., 1992, "Method for Selecting Master DOF in Dynamic Substructuring Using the Guyan Condensation Method," *Computers and Structures*, **45**, pp. 941–946.
- [46] Brahmi, K., Bouhaddi, N., and Fillod, R., 1995, "Reduction of the Junction Degrees of Freedom Before Assembly in Dynamic Substructuring," *Acoustics, Vibrations, and Rotating Machines*, American Society of Mechanical Engineers, New York, NY, **DE-84-2**, pp. 699–708.
- [47] Bouhaddi, N., and Fillod, R., 1996, "Model Reduction by a Simplified Variant of Dynamic Condensation," *Journal of Sound and Vibration*, **191**, pp. 233–250.
- [48] Bouhaddi, N., and Fillod, R., 1996, "Substructuring by a Two Level Dynamic Condensation Method," *Computers and Structures*, **60**, pp. 403–409.
- [49] Brahmi, K., and Bouhaddi, N., 1997, "Improved Component Mode Synthesis Based on Double Condensation Method," *Proceedings of the 15th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **2**, pp. 1469–1475.
- [50] Bouhaddi, N., and Lombard, J. P., 2000, "Improved Free-Interface Substructures Representation Method," *Computers and Structures*, **77**, pp. 269–283.
- [51] Balmès, E., 1996, "Optimal Ritz Vectors for Component Mode Synthesis Using the Singular Value Decomposition," *AIAA Journal*, **34**, pp. 1256–1260.
- [52] Balmès, E., 1996, "Use of Generalized Interface Degrees of Freedom in Component Mode Synthesis," *Proceedings of the 14th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 204–210.
- [53] Balmès, E., 1997, "Component Mode Synthesis for Families of Reduced Finite Element Models," *New Advances in Modal Synthesis of Large Structures*, A. A. Balkema, Rotterdam, Netherlands, pp. 41–52.
- [54] Balmès, E., and Billet, L., 2001, "Using Expansion and Interface Reduction to Enhance Structural Modification Methods," *Proceedings of the 19th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 615–621.
- [55] Ohayon, R., Sampaio, R., and Soize, C., 1997, "Dynamic Substructuring of Damped Structures Using Singular Value Decomposition," *Journal of Applied Mechanics*, **64**, pp. 292–298.

- [56] Rixen, D., Farhat, C., and Geradin, M., 1998, "A Two-Step, Two-Field Hybrid Method for the Static and Dynamic Analysis of Substructure Problems with Conforming and Non-Conforming Interfaces," *Computer Methods in Applied Mechanics and Engineering*, **154**, pp. 229–264.
- [57] Rixen, D. J., 2002, "Force Modes for Reducing the Interface Between Substructures," *Proceedings of IMAC-XX: A Conference on Structural Dynamics*, Society for Experimental Mechanics, Bethel, CT, **2**.
- [58] Aoyama, Y., and Yagawa, G., 2001, "Component Mode Synthesis for Large-Scale Structural Eigenanalysis," *Computers and Structures*, **79**, pp. 605–615.
- [59] Aoyama, Y., and Yagawa, G., 2001, "Large-Scale Eigenvalue Analysis of Structures Using Component Mode Synthesis," *JSME International Journal, Series A: Solid Mechanics and Material Engineering*, **44**, pp. 631–640.
- [60] Jacobsen, K. P., 1983, "Fully Integrated Superelements: A Database Approach to Finite Element Analysis," *Computers and Structures*, **16**, pp. 307–315.
- [61] Elwi, A. E., and Murray, D. W., 1985, "Skyline Algorithms for Multilevel Substructure Analysis," *International Journal for Numerical Methods in Engineering*, **21**, pp. 465–479.
- [62] Prakash, B. G., and Prabhu, M. S. S., 1986, "Reduction Techniques in Dynamic Substructures for Large Problems," *Computers and Structures*, **22**, pp. 539–552.
- [63] Leung, A. Y. T., 1989, "Multilevel Dynamic Substructures," *International Journal for Numerical Methods in Engineering*, **28**, pp. 181–191.
- [64] Mourelatos, Z. P., 2000, "An Efficient Crankshaft Dynamic Analysis Using Substructuring with Ritz Vectors," *Journal of Sound and Vibration*, **238**, pp. 495–527.
- [65] Bennighof, J. K., and Kim, C. K., 1992, "Adaptive Multi-Level Substructuring Method for Efficient Modeling of Complex Structures," *Proceedings of the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, American Institute of Aeronautics and Astronautics, Reston, VA, pp. 1631–1639.
- [66] Bennighof, J. K., Kaplan, M. F., and Muller, M. B., 1997, "Frequency Response Analysis Using Adaptive Multi-Level Substructuring," *Proceedings of the 15th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 64–70.
- [67] Bennighof, J. K., and Kaplan, M. F., 1998, "Frequency Window Implementation of Adaptive Multi-Level Substructuring," *Journal of Vibration and Acoustics*, **120**, pp. 409–418.
- [68] Bennighof, J. K., and Kaplan, M. F., 1998, "Frequency Sweep Analysis Using Multi-Level Substructuring, Global Modes and Iteration," *Proceedings of the 39th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, American Institute of Aeronautics and Astronautics, Reston, VA, **4**, pp. 2567–2575.

- [69] Bennighof, J. K., 1999, "Vibroacoustic Frequency Sweep Analysis Using Automated Multi-Level Substructuring," *Proceedings of the 40th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, American Institute of Aeronautics and Astronautics, Reston, VA, **1**, pp. 422–427.
- [70] Bennighof, J. K., Kaplan, M. F., and Muller, M. B., 1999, "Reducing NVH Analysis Burden Using Automated Multi-Level Substructuring," *Proceedings of the 17th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 372–377.
- [71] Bennighof, J. K., Kaplan, M. F., Muller, M. B., and Kim, M., 2000, "Meeting the NVH Computational Challenge: Automated Multi-Level Substructuring," *Proceedings of the 18th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 909–915.
- [72] Mace, B. R., and Shorter, P. J., 2000, "Energy Flow Models from Finite Element Analysis," *Journal of Sound and Vibration*, **233**, pp. 369–389.
- [73] Mace, B. R., and Shorter, P. J., 2001, "A Local Modal/Perturbational Method for Estimating Frequency Response Statistics of Built-Up Structures With Uncertain Properties," *Journal of Sound and Vibration*, **242**, pp. 793–811.
- [74] Tan, Y.-C., Castanier, M. P., and Pierre, C., 2001, "Approximation of Power Flow Between Two Coupled Beams Using Statistical Energy Methods," *Journal of Vibration and Acoustics*, **123**, pp. 510–523.
- [75] Zhang, G., Castanier, M. P., Pierre, C., and Mourelatos, Z. P., 2003, "Vibration and Power Flow Analysis of a Vehicle Structure Using Characteristic Constraint Modes," *Proceedings of the SAE Noise and Vibration Conference and Exhibition*, Society of Automotive Engineers, Troy, MI. Submitted for publication.
- [76] Woodhouse, J., 1981, "An Approach to the Theoretical Background of Statistical Energy Analysis Applied to Structural Vibration," *Journal of the Acoustical Society of America*, **69**, pp. 1695–1709.
- [77] Keane, A. J., and Price, W. G., 1987, "Statistical Energy Analysis of Strongly Coupled Systems," *Journal of Sound and Vibration*, **117**, pp. 363–386.
- [78] Langley, R. S., 1989, "A General Derivation of the Statistical Energy Analysis Equations for Coupled Dynamic Systems," *Journal of Sound and Vibration*, **135**, pp. 499–508.
- [79] Langley, R. S., 1990, "A Derivation of the Coupling Loss Factors Used in Statistical Energy Analysis," *Journal of Sound and Vibration*, **141**, pp. 207–219.
- [80] Fahy, F. J., 1994, "Statistical Energy Analysis: A Critical Overview," *Philosophical Transactions of the Royal Society of London, Series A*, **346**.
- [81] Mace, B. R., 1994, "On the Statistical Energy Analysis Hypothesis of Coupling Power Proportionality and Some Implications of its Failure," *Journal of Sound and Vibration*, **178**, pp. 95–112.

- [82] Dowell, E. H., and Kubota, Y., 1985, "Asymptotic Modal Analysis and Statistical Energy Analysis of Dynamical Systems," *Journal of Applied Mechanics*, **52**, pp. 949–957.
- [83] Kubota, Y., and Dowell, E. H., 1986, "Experimental Investigation of Asymptotic Modal Analysis for a Rectangular Plate," *Journal of Sound and Vibration*, **106**, pp. 203–216.
- [84] Dowell, E. H., 1987, "Classical Modal Analysis, Asymptotic Modal Analysis, and Statistical Energy Analysis," *Statistical Energy Analysis*, American Society of Mechanical Engineers, New York, NY, NCA-3, pp. 25–26.
- [85] Kubota, Y., Dionne, H. D., and Dowell, E. H., 1988, "Asymptotic Modal Analysis and Statistical Energy Analysis of an Acoustic Cavity," *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, **110**, pp. 371–376.
- [86] Peretti, L. F., and Dowell, E. H., 1992, "Asymptotic Modal Analysis of a Rectangular Acoustic Cavity Excited by Wall Vibration," *AIAA Journal*, **30**, pp. 1191–1198.
- [87] Doherty, S. M., and Dowell, E. H., 1994, "Experimental Study of Asymptotic Modal Analysis Applied to a Rectangular Plate With Concentrated Masses," *Journal of Sound and Vibration*, **170**, pp. 671–681.
- [88] Reynolds, R. R., and Dowell, E. H., 1998, "The Transient Response of Structures Using Asymptotic Modal Analysis," *Journal of Applied Mechanics*, **65**, pp. 258–265.
- [89] Sestieri, A., 2001, "Vibroacoustics Beyond 2000: Looking for the Sound of Silence," *Structural Dynamics @ 2000: Current Status and Future Directions*, Research Studies Press Ltd., Baldock, Hertfordshire, England.
- [90] Langley, R. S., 1992, "A Wave Intensity Technique for the Analysis of High Frequency Vibrations," *Journal of Sound and Vibration*, **159**, pp. 483–502.
- [91] Langley, R. S., and Bercin, A. N., 1994, "Wave Intensity Analysis of High Frequency Vibrations," *Philosophical Transactions of the Royal Society of London, Series A*, **346**, pp. 489–499.
- [92] Keane, A. J., and Price, W. G., 1989, "Statistical Energy Analysis of Periodic Structures," *Proceedings of the Royal Society of London, Series A*, **423**, pp. 331–360.
- [93] Pierre, C., Castanier, M. P., and Choi, S. B., 1997, "On Developing New Statistical Energy Methods for the Analysis of Vibration Transmission in Complex Vehicle Structures," *Mechanics of Structures and Machines*, **25**, pp. 87–101.
- [94] Sablik, M. J., Beissner, R. E., Silvus, H. S., and Miller, M. L., 1985, "Statistical Energy Analysis, Structural Resonances, and Beam Networks," *Journal of the Acoustical Society of America*, **77**, pp. 1038–1045.
- [95] Fredö, C. R., 1997, "A SEA-Like Approach for the Derivation of Energy Flow Coefficients with a Finite Element Model," *Journal of Sound and Vibration*, **199**, pp. 645–666.

- [96] Simmons, C., 1991, "Structure-Borne Sound Transmission Through Plate Junctions and Estimates of SEA Coupling Loss Factors Using the Finite Element Method," *Journal of Sound and Vibration*, **144**, pp. 215–227.
- [97] Maxit, L., and Guyader, J.-L., 2001, "Estimation of SEA Coupling Loss Factors Using a Dual Formulation and FEM Modal Information, Part I: Theory," *Journal of Sound and Vibration*, **239**, pp. 907–930.
- [98] Maxit, L., and Guyader, J.-L., 2001, "Estimation of SEA Coupling Loss Factors Using a Dual Formulation and FEM Modal Information, Part II: Numerical Applications," *Journal of Sound and Vibration*, **239**, pp. 931–948.
- [99] Yan, H., Parrett, A., and Nack, W., 2000, "Statistical Energy Analysis by Finite Elements for Middle Frequency Vibration," *Finite Elements in Analysis and Design*, **35**, pp. 297–304.
- [100] Zhang, Q. J., and Sainsbury, M. G., 1999, "Energy Flow Method for Strongly Coupled Systems," *Proceedings of the 1999 17th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **2**, pp. 1839–1845.
- [101] Guyader, J. L., Boisson, C., and Lesueur, C., 1982, "Energy Transmission in Finite Coupled Plates, Part I: Theory," *Journal of Sound and Vibration*, **81**, pp. 81–92.
- [102] Boisson, C., Guyader, J. L., Millot, P., and Lesueur, C., 1982, "Energy Transmission in Finite Coupled Plates, Part II: Application to an L Shaped Structure," *Journal of Sound and Vibration*, **81**, pp. 93–105.
- [103] Lyon, R. H., 1995, "SEA, Power Flow, and Energy Accountancy," SAE Paper 951303.
- [104] Belov, V. D., Rybak, S. A., and Tartakovskii, B. D., 1977, "Propagation of Vibrational Energy in Absorbing Structures," *Journal of Soviet Physics Acoustics*, **23**, pp. 115–119. In Russian.
- [105] Buvailo, L. E., and Ionov, A. E., 1980, "Application of the Finite Element Method to the Investigation of the Vibroacoustical Characteristics of Structures at High Audio Frequencies," *Journal of Soviet Physics Acoustics*, **26**, pp. 277–279. In Russian.
- [106] Belyaev, A. K., and Palmov, V. A., 1986, "Integral Theories of Random Vibration of Complex Structures," *Random Vibration — Status and Recent Developments* (Elishakoff, I., and Lyon, R. H., eds.), Elsevier, Amsterdam, pp. 19–38.
- [107] Belyaev, A. K., 1994, "Vibrational Conductivity Approach to High-Frequency Dynamics," *Nuclear Engineering and Design*, **150**, pp. 281–286.
- [108] Belyaev, A. K., 1999, "Modelling of the Local Dynamics of Structural Members by Means of the Vibrational Conductivity Approach," *Structural Dynamics: Proceedings of the Fourth European Conference on Structural Dynamics*, Balkema, Rotterdam, Netherlands, **1**, pp. 139–144.

- [109] Nefske, D. J., and Sung, S. H., 1989, "Power Flow Finite Element Analysis of Dynamic Systems: Basic Theory and Application to Beams," *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, **111**, pp. 94–100.
- [110] Wohlever, J. C., and Bernhard, R. J., 1992, "Mechanical Energy Flow Models of Rods and Beams," *Journal of Sound and Vibration*, **153**, pp. 1–19.
- [111] Bouthier, O. M., and Bernhard, R. J., 1992, "Models of Space-Averaged Energetics of Plates," *AIAA Journal*, **30**, pp. 616–623.
- [112] Bouthier, O. M., and Bernhard, R. J., 1995, "Simple Models of Energy Flow in Vibrating Membranes," *Journal of Sound and Vibration*, **182**, pp. 129–147.
- [113] Bouthier, O. M., and Bernhard, R. J., 1995, "Simple Models of the Energetics of Transversely Vibrating Plates," *Journal of Sound and Vibration*, **182**, pp. 149–164.
- [114] Palmer, J. D., Williams, E. J., and Fox, C. H. J., 1992, "High Frequency Power Flow in Structures," *Proceedings of the 10th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 701–707.
- [115] Palmer, J. D., Williams, E. J., and Fox, C. H. J., 1993, "Power Flow in Complex Structures," *Proceedings of the 11th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 115–121.
- [116] Stiehl, A. L., 1996, "Power Flow Analysis of Beam Members With Multiple Wave Types," *Finite Elements in Analysis and Design*, **21**, pp. 253–264.
- [117] Moens, I., Vandepitte, D., and Sas, P., 1997, "Energy Flow Analysis of a Beam Network: Comparison Between Numerical and Experimental Results," *Proceedings of the 15th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**.
- [118] Langley, R. S., 1995, "On the Vibrational Conductivity Approach to High Frequency Dynamics for Two-Dimensional Structural Components," *Journal of Sound and Vibration*, **182**, pp. 637–657.
- [119] Carcaterra, A., and Sestieri, A., 1995, "Energy Density Equations and Power Flow in Structures," *Journal of Sound and Vibration*, **188**, pp. 269–282.
- [120] Sestieri, A., and Carcaterra, A., 1995, "An Envelope Energy Model for High Frequency Dynamic Structures," *Journal of Sound and Vibration*, **188**, pp. 283–295.
- [121] Carcaterra, A., and Sestieri, A., 1997, "Complex Envelope Displacement Analysis: A Quasi-Static Approach to Vibrations," *Journal of Sound and Vibration*, **201**, pp. 205–233.
- [122] Xing, J.-T., and Price, W. G., 1999, "A Power-Flow Analysis Based on Continuum Dynamics," *Proceedings of the Royal Society of London, Series A*, **455**, pp. 401–436.

- [123] Vlahopoulos, N., Garza-Rios, L. O., and Mollo, C., 1999, "Numerical Implementation, Validation, and Marine Applications of an Energy Finite Element Formulation," *Journal of Ship Research*, **43**, pp. 143–156.
- [124] Vlahopoulos, N., Zhao, X., and Allen, T., 1999, "An Approach for Evaluating Power Transfer Coefficients for Spot-Welded Joints in an Energy Finite Element Formulation," *Journal of Sound and Vibration*, **220**, pp. 135–154.
- [125] Vlahopoulos, N., and Zhao, X., 1999, "Basic Development of Hybrid Finite Element Method for Midfrequency Structural Vibrations," *AIAA Journal*, **37**, pp. 1495–1505.
- [126] Zhao, X., and Vlahopoulos, N., 2000, "A Hybrid Finite Element Formulation for Mid-Frequency Analysis of Systems With Excitation Applied on Short Members," *Journal of Sound and Vibration*, **237**, pp. 181–202.
- [127] Vlahopoulos, N., and Zhao, X., 2001, "An Investigation of Power Flow in the Mid-Frequency Range for Systems of Co-Linear Beams Based on a Hybrid Finite Element Formulation," *Journal of Sound and Vibration*, **242**, pp. 445–473.
- [128] Bitsie, F., and Bernhard, R. J., 1998, "Sensitivity Calculations for Structural-Acoustic EFEM Predictions," *Noise Control Engineering Journal*, **46**, pp. 91–96.
- [129] Bernhard, R. J., and Huff, Jr., J. E., 1999, "Structural-Acoustic Design At High Frequency Using the Energy Finite Element Method," *Journal of Vibration and Acoustics*, **121**, pp. 295–301.
- [130] Wang, S., and Bernhard, R. J., 2002, "Theory and Applications of a Simplified Energy Finite Element Method and its Similarity to SEA," *Noise Control Engineering Journal*, **50**, pp. 63–72.
- [131] Lase, Y., Ichchou, M. N., and Jezequel, L., 1996, "Energy Flow Analysis of Bars and Beams: Theoretical Formulations," *Journal of Sound and Vibration*, **192**, pp. 281–305.
- [132] Ichchou, M. N., and Jezequel, L., 1996, "Comments on Simple Models of the Energy Flow in Vibrating Membranes and on Simple Models of the Energetics of Transversely Vibrating Plates," *Journal of Sound and Vibration*, **195**, pp. 679–685.
- [133] Ichchou, M. N., Le Bot, A., and Jezequel, L., 1997, "Energy Models of One-Dimensional, Multi-Propagative Systems," *Journal of Sound and Vibration*, **201**, pp. 535–554.
- [134] Le Bot, A., 1998, "A Vibroacoustic Model for High Frequency Analysis," *Journal of Sound and Vibration*, **211**, pp. 537–554.
- [135] Jezequel, L., and Ichchou, M. N., 2001, "New Strategies in Structural Dynamics for the Non Modal Domain," *Proceedings of the 19th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **2**, pp. 1092–1098.
- [136] Cotoni, V., Le Bot, A., and Jezequel, L., 2002, "High-Frequency Radiation of L-Shaped Plates by a Local Energy Flow Approach," *Journal of Sound and Vibration*, **250**, pp. 431–444.

- [137] Viktorovitch, M., Thouverez, F., and Jezequel, L., 1999, "A New Random Boundary Element Formulation Applied to High Frequency Phenomena," *Journal of Sound and Vibration*, **223**, pp. 273–296.
- [138] Viktorovitch, M., Thouverez, F., and Jezequel, L., 2001, "An Integral Formulation With Random Parameters Adapted to the Study of the Vibrational Behaviour of Structures in the Middle- and High-Frequency Field," *Journal of Sound and Vibration*, **247**, pp. 431–452.
- [139] Girard, A., and Defosse, H., 1990, "Frequency Response Smoothing, Matrix Assembly and Structural Paths: A New Approach for Structural Dynamics Up to High Frequencies," *Journal of Sound and Vibration*, **137**, pp. 53–68.
- [140] Girard, A., and Defosse, H., 1993, "Frequency Response Smoothing and Structural Path Analysis: Application to Beam Trusses," *Journal of Sound and Vibration*, **165**, pp. 165–170.
- [141] Girard, A., Dupuis, P. E., and Bugeat, L. P., 1997, "Dynamic Analysis in Medium Frequency Domain by Frequency Smoothing and Structural Path Analysis," *New Advances in Modal Synthesis of Large Structures*, A. A. Balkema, Rotterdam, Netherlands, pp. 301–311.
- [142] Soize, C., Hutin, P. M., Desanti, A., David, J. M., and Chabas, F., 1986, "Linear Dynamic Analysis of Mechanical Systems in the Medium Frequency Range," *Computers and Structures*, **23**, pp. 605–637.
- [143] Soize, C., 1993, "A Model and Numerical Method in the Medium Frequency Range for Vibroacoustic Predictions Using the Theory of Structural Fuzzy," *Journal of the Acoustical Society of America*, **94**, pp. 849–865.
- [144] Soize, C., 1998, "Estimation of the Fuzzy Substructure Model Parameters Using the Mean Power Flow Equation of the Fuzzy Structure," *Journal of Vibration and Acoustics*, **120**, pp. 279–286.
- [145] Soize, C., and Bjaoui, K., 2000, "Estimation of Fuzzy Structure Parameters for Continuous Junctions," *Journal of the Acoustical Society of America*, **107**, pp. 2011–2020.
- [146] Pierce, A. D., Sparrow, V. W., and Russell, D. A., 1995, "Fundamental Structural-Acoustic Idealizations for Structures with Fuzzy Internals," *Journal of Vibration and Acoustics*, **117**, pp. 339–348.
- [147] Sparrow, V. W., Russell, D. A., and Rochat, J. L., 1994, "Implementation of Discrete Fuzzy Structure Models in Mathematica," *International Journal for Numerical Methods in Engineering*, **37**, pp. 3005–3014.
- [148] Rochat, J. L., and Sparrow, V. W., 1997, "Incorporating Compressional and Shear Wave Types into Fuzzy Structure Models for Plates," *Proceedings of the 1995 Design Engineering Technical Conferences*, American Society of Mechanical Engineers, New York, NY, **DE-84-2**, pp. 247–252.

- [149] Sparrow, V., and Buehrle, R., 2001, "Fuzzy Structures Analysis of Aircraft Panels in NAS-TRAN," *Proceedings of the 42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics Conference and Exhibit*, American Institute of Aeronautics and Astronautics, 2, pp. 1032–1041.
- [150] Ruckman, C. E., and Feit, D., 1995, "A Tutorial on Soize's Method for Stochastic Modeling in Structural Acoustics (Fuzzy Structures Analysis)," *Proceedings of the 1995 Design Engineering Technical Conferences*, American Society of Mechanical Engineers, New York, NY, **DE-84-2**, pp. 241–246.
- [151] Soize, C., 1998, "Trends in Modeling of Structural-Acoustics Systems with Structural Complexity in Low- and Medium-Frequency Ranges," *Proceedings of the 16th International Congress on Acoustics*.
- [152] Vasudevan, R., and Liu, Y. N., 1991, "Application of Time Integration and Transform Techniques to Scattering Problems," *Structural Acoustics*, American Society of Mechanical Engineers, New York, NY, **NCA-12/AMD-128**, pp. 35–40.
- [153] Liu, W. K., Zhang, Y., and Ramirez, M. R., 1991, "Multiple Scale Finite Element Methods," *International Journal for Numerical Methods in Engineering*, **32**, pp. 969–990.
- [154] Sparrow, V. W., 1999, "Mid-Frequency Range Finite Element (MFR-FE) Approach for Structural Vibrations and Interior Noise," *Proceedings of the 40th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, *AIAA Paper 99-1855*, American Institute of Aeronautics and Astronautics.
- [155] Savin, E., 2002, "Midfrequency Vibrations of a Complex Structure: Experiments and Comparison with Numerical Simulations," *AIAA Journal*, **40**, pp. 1876–1884.
- [156] Soize, C., 1998, "Reduced Models in the Medium Frequency Range for General Dissipative Structural-Dynamics Systems," *European Journal of Mechanics, A/Solids*, **17**, pp. 657–685.
- [157] Soize, C., 1999, "Reduced Models for Computational Structural Acoustics in the Medium-Frequency Range," *Proceedings of the ASME Noise Control and Acoustics Division — 1999*, American Society of Mechanical Engineers, New York, NY, **NCA-26**, pp. 479–485.
- [158] Soize, C., and Mziou, S., "Dynamic Substructuring in the Medium-Frequency Range," ON-ERA Report TP 2000-210, Chatillon, France, 2000.
- [159] Pinnington, R. J., and White, R. G., 1981, "Power Flow Through Machine Isolators to Resonant and Non-Resonant Beams," *Journal of Sound and Vibration*, **75**, pp. 179–197.
- [160] Cuschieri, J. M., 1990, "Structural Power-Flow Analysis Using a Mobility Approach of an L-Shaped Plate," *Journal of the Acoustical Society of America*, **87**, pp. 1159–1165.
- [161] Cuschieri, J. M., 1990, "Vibration Transmission Through Periodic Structures Using a Mobility Power Flow Approach," *Journal of Sound and Vibration*, **143**, pp. 65–74.

- [162] McCollum, M. D., and Cuschieri, J. M., 1990, "Thick Plate Bending Wave Transmission Using a Mobility Power Flow Approach," *Journal of the Acoustical Society of America*, **88**, pp. 1472–1479.
- [163] Cuschieri, J. M., and McCollum, M. D., 1996, "In-Plane and Out-Of-Plane Waves' Power Transmission Through an L-Plate Junction Using the Mobility Power Flow Approach," *Journal of the Acoustical Society of America*, **100**, pp. 857–870.
- [164] Ladevèze, P., 1996, "Une Nouvelle Approche pour le Calcul des Vibrations Moyennes Fréquences," *Comptes Rendus de l'Académie des Sciences, Série IIb*, **322**, pp. 849–856.
- [165] Ladevèze, P., Arnaud, L., Rouch, P., and Blanze, C., 2001, "The Variational Theory of Complex Rays for the Calculation of Medium-Frequency Vibrations," *Engineering Computations*, **18**, pp. 193–214.
- [166] Hasselman, T. K., Anderson, M. C., and Gan, W., 1998, "Principal Components Analysis for Nonlinear Model Correlation, Updating and Uncertainty Evaluation," *Proceedings of the 16th International Modal Analysis Conference*, Society for Experimental Mechanics, Bethel, CT, **1**, pp. 644–651.
- [167] Hasselman, T., 2001, "Quantification of Uncertainty in Structural Dynamic Models," *Journal of Aerospace Engineering*, **14**, pp. 158–165.
- [168] Lu, L., 1990, "Dynamic Substructuring by FEA/SEA," *Vehicle Noise*, American Society of Mechanical Engineers, New York, NY, NCA-9, pp. 9–12.
- [169] Belyaev, A. K., 1992, "Dynamic Simulation of High-Frequency Vibration of Extended Complex Structures," *Mechanics of Structures and Machines*, **20**, pp. 155–168.
- [170] Belyaev, A. K., 1993, "High-Frequency Vibration of Extended Complex Structures," *Probabilistic Engineering Mechanics*, **8**, pp. 15–24.
- [171] Shorter, P., and Langley, R., 2000, "The Spatial Correlation of Vibrational Wavefields and Their Application to Mid-Frequency Structural-Acoustics," *Proceedings of the International Conference on Noise and Vibration Pre-Design and Characterisation Using Energy Methods*, French Acoustical Society, Paris, France.
- [172] Shorter, P. J., Gardner, B. K., and Patil, A., 2001, "Resound — A Full Spectrum Modelling Method," *Proceedings of the ASME Noise Control and Acoustics Division — 2001*, American Society of Mechanical Engineers, New York, NY, NCA-28, pp. 153–160.
- [173] Vibro-Acoustic Sciences, Inc. Resound research overview and examples at web site, <http://www.vasci.com/research/resound/>.
- [174] Lyon, R. H., 1995, "Statistical Energy Analysis and Structural Fuzzy," *Journal of the Acoustical Society of America*, **97**, pp. 2878–2881.
- [175] Mace, B. R., 1992, "Power Flow Between Two Continuous One-Dimensional Subsystems: A Wave Solution," *Journal of Sound and Vibration*, **154**, pp. 289–319.

- [176] Mace, B. R., 1992, "The Statistics of Power Flow Between Two Continuous One-Dimensional Subsystems," *Journal of Sound and Vibration*, **154**, pp. 321–341.
- [177] Cudney, H., 2001, "Discussions and Debates at SD2000," *Structural Dynamics @ 2000: Current Status and Future Directions*, Research Studies Press Ltd., Hertfordshire, England, pp. 413–427.
- [178] Castanier, M. P., Óttarsson, G., and Pierre, C., 1997, "A Reduced-Order Modeling Technique for Mistuned Bladed Disks," *Journal of Vibration and Acoustics*, **119**, pp. 439–447.
- [179] Bladh, R., Castanier, M. P., and Pierre, C., 2001, "Component-Mode-Based Reduced Order Modeling Techniques for Mistuned Bladed Disks, Part I: Theoretical Models," *Journal of Engineering for Gas Turbines and Power*, **123**, pp. 89–99.
- [180] Bladh, R., Castanier, M. P., and Pierre, C., 2001, "Component-Mode-Based Reduced Order Modeling Techniques for Mistuned Bladed Disks, Part II: Application," *Journal of Engineering for Gas Turbines and Power*, **123**, pp. 100–108.
- [181] Yang, M.-T., and Griffin, J. H., 2001, "A Reduced-Order Model of Mistuning Using A Subset of Nominal System Modes," *Journal of Engineering for Gas Turbines and Power*, **123**, pp. 893–900.
- [182] Petrov, E. P., Sanliturk, K. Y., and Ewins, D. J., 2002, "A New Method for Dynamic Analysis of Mistuned Bladed Disks Based on the Exact Relationship Between Tuned and Mistuned Systems," *Journal of Engineering for Gas Turbines and Power*, **124**, pp. 586–597.
- [183] Box, G. E. P., 1979, "Robustness in the Strategy of Scientific Model Building," *Robustness in Statistics*, Academic Press, New York, NY, pp. 201–236.

Guangquan Wu

1437 Natalie Lane, Apt.#308

Ann Arbor, MI 48105

Tel&FAX: (734)332-8793

E-mail wugq@umich.com

Education

Ph.D in Mechanical Engineering, Thesis title: The dynamic simulation and systemic optimization on the valve gear of internal combustion engine , Jilin University of Technology, China. Nov. 1986 - Jul. 1991

M.S. in Mechanical Engineering, Thesis title: The study on the flow and combustion in SI engine by using high speed photography, Jilin University of Technology, China. Aug. 1984 - Oct. 1986

B.S. in Mechanical Engineering, Jilin University of Technology, China. Aug. 1980 - Jul. 1984

Field of Knowledge

Thermodynamics, Fluid Mechanics, Heat Transfer, FEM, Multi-body Dynamics, Mechanism Dynamics, Optimization, CFD, CAD, Engine/Powertrain, Vehicle System, Testing and Experimentation.

Career Experiences

Automotive Lab in University of Michigan. Aug. 2001 - Present

Visiting Assistant Research Scientist, engaged in two projects. One is among the project(Ford DUST) "simulation based advanced next generation diesel technology" and focuses on cooling system simulation and will evaluate the impact of electrical coolant pump on engine performance and emissions; Another one is among the project(GM DUST) "effect of manufacturing variations on the engine performance and emissions" and focuses on the valvetrain assembly and manifold surface roughness. GT-Suite, including GT-Cool, GT-Power and GT-Vtrain are the main tools used in these projects. Other optimization and control codes, such as iSIGHT and MATLAB/SIMULINK are also used.

CD-adapco Japan Co., LTD, Japan. Nov. 1997 - May 2000

Lead support engineer in GT-SUITE products for Japanese customers, and also for Chinese clients.

Responsible for GT-power's training, daily support, consultation and communication with Gamma Technologies Inc. in U.S., as well as the support at the GT-SUITE coupled with Star-CD and SIMULINK.

Tsinghua University, China. Dec. 1996 - Oct. 1997

Associate Professor, Director of education and research lab in internal combustion engine. Responsible for organizing the education and research affairs related to engine, and also giving the lectures and doing the adviser of undergraduate and graduate student. Research at the simulating and experimental study on the intake and exhaust manifold of MPISI engine.

Isuzu Motors Co., LTD., Japan. Feb. 1995 - Nov. 1996

Visiting Scholar, Responsible for the simulation of the flow and spray in cylinder of DISI engine by using CFD code Star-CD and the simulation of engine performance by using WAVE.

Tsinghua University, China. Aug. 1991 - Jan. 1995

Lecturer, Responsible for the two undergraduate courses: Engine design, CAD program(AutoCAD).

Mainly engaged in the research project: "The wear-resistance study on the engine used in the desert vehicle" The key project of National 85 Plan. By cooperating with the Bureau of Desert Petroleum Exploration, which belongs to Ministry of Petroleum in China, and the air cleaner and engine parts makers, we firstly advanced a technique for analyzing and measuring the air cleaner performance at the real operating state of vehicle. Furthermore, by improving the air cleaner, we solved the fast worn problem of desert vehicle engine under the particular environment of the biggest desert Taklamakan in China. For the excellent work in this research we were awarded by the Ministry of Machinery in China.

Jilin University of Technology, China. Nov. 1986 - Jul. 1991

Graduate student for Ph.D., The work includes: the design method of cam profile by using finite terms of Fourier Series, the FEM and Flexible Multi-body dynamic models of valve gear, the systemic optimization model and experimental study on the dynamic performance and noise of valve gear.

Jilin University of Technology, China. Aug. 1984 - Oct. 1986

Graduate student for Master degree, The work includes: the construction of experimental engine with the lengthened and transparent top piston, the experimental study on the flow and combustion in cylinder by using high speed photography.

Awards

1992 Award for Excellent Teacher in Tsinghua university.

1994 Award for Excellent Research given by Ministry of Machinery in China.

The research subject is: "A Technique for Measuring and Analyzing the Performance of Air-cleaner during the Vehicle's Real Operating Condition".

Related Publications

XinMing Wu,..., Guangquan Wu "The Correction Method on the Measurement Error of Cam Lift Caused by the Eccentric Rotation of Cam Shaft and the Creation of Cam Profile by Fourier Series." Proceedings of annual conference of the society of Chinese internal combustion engine . May, 1984.

Guangquan Wu, Huizhen Li "An Effective Method of Solving Multi-body Dynamic Equation and Its Application in the Dynamic Analysis of Valve Gear in Internal Combustion Engine". Proceedings of ICDVC (International Conference on Dynamics, Vibration and Control) July, 1990, China, Beijing.

Guangquan Wu, Huizhen Li, Xiaokuan Lu "The Optimum Design of Cam Profile of Internal Combustion Engine" Journal of Jilin University of Technology of China Vol.2 1990.

Guangquan Wu, Huizhen Li, Xiaokuan Lu "Research on Valve Gear of Internal Combustion Engine by Flexible Multi-body Dynamics" Journal of Internal Combustion Engine of China Vol.1 1992.

Junbing Le, Huizhen Li, Guangquan Wu "Dynamic Analysis of Valve Gear in Internal Combustion Engine by FEM" Journal of Automobile Engineering of China Vol. 1 1994.

Guangquan Wu, et al. "The Investigation and Analysis on the Desert Environment and Air-cleaner Used in Desert Trucks" Proceedings of annual conference of the society of Chinese internal combustion engine. Aug., 1994.

Junbing Le,...,Guangquan Wu "The Study on Valve Gear of Overhead Valve and Overhead Camshaft Drive in Internal Combustion Engine by FEM and Experiment" Journal of Automobile Technology of China Vol.2 1995.

Hua Zhang, Guangquan Wu, etc. "The Measurement and Study Method on the Real Operating Effectiveness of Air Cleaner in Desert Truck" Journal of Internal Combustion Engine Engineering of China Vol. 3 1995.

Guangquan Wu, Wei Li, Renjun Zhuang "The experimental and simulating study on the intake and exhaust manifold in MPI SI engine" Proceedings of annual conference of the society of Chinese internal combustion engine. Oct. 1997.

Shi Huang, ..., Guangquan Wu "Research on Reducing Engine Wear of Desert Vehicle" Journal of Tsinghua University, Vol. 37 n8 1997, P87-90